UNIVERSITY OF SOUTHAMPTON

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School of Electronics and Computer Science

Quantum Error Correction Codes

by

Zunaira Babar

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SUPERVISORS

Dr Soon Xin Ng

and Professor Lajos Hanzo

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ABSTRACT

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Quantum parallel processing techniques are capable of solving certain complex problems at a substantially lower complexity than their classical counterparts. From the perspective of telecommunications, this quantum-domain parallel processing provides a plausible solution for achieving full-search based multi-stream detection, which is vital for future gigabit-wireless systems. The peculiar laws of quantum mechanics have also spurred interest in the absolutely-secure quantum-based communication systems. Unfortunately, quantum decoherence imposes a hitherto insurmountable impairment on the practical implementation of quantum computation as well as on quantum communication systems, which may be overcome with the aid of efficient error correction codes. In this thesis, we design error correction codes for the quantum domain, which is an intricate journey from the realm of classical channel coding theory to that of the Quantum Error Correction Codes (QECCs).

Since quantum-based communication systems are capable of supporting the transmission of both classical and quantum information, we initially focus our attention on the code design for entanglement-assisted classical communication over the quantum depolarizing channel. We conceive an EXtrinsic Information Transfer (EXIT) chart aided near-capacity classical-quantum code design, which invokes a classical Irregular Convolutional Code (IRCC) and a Unity Rate Code (URC) in conjunction with our proposed soft-decision aided SuperDense Code (SD). Hence, it is referred to as an 'IRCC-URC-SD' arrangement. The proposed scheme is intrinsically amalgamated both with 2-qubit as well as 3-qubit SD coding protocols and it is benchmarked against the corresponding entanglement-assisted classical capacity. Since the IRCC-URC-SD scheme is a bit-based design, it incurs a capacity loss. As a further advance, we design a symbol-based concatenated code design, referred to as a symbol-based 'CC-URC-SD', which relies on a single-component classical Convolutional Code (CC). Additionally, for the sake of reducing the associated decoding complexity, we also investigate the impact of the constraint length of the convolutional code on the achievable performance.

Our initial designs, namely IRCC-URC-SD and CC-URC-SD, exploit redundancy in the classical domain. By contrast, QECCs relying on the quantum-domain redundancy are indispensable for conceiving a quantum communication system supporting the transmission of quantum information and also for quantum computing. Therefore, we next provide insights into the transformation from the

family of classical codes to the class of quantum codes known as 'Quantum Stabilizer Codes' (QSC), which invoke the classical syndrome decoding. Particularly, we detail the underlying quantum-to-classical isomorphism, which facilitates the design of meritorious families of QECCs from the known classical codes. We further study the syndrome decoding techniques operating over classical channels, which may be exploited for decoding QSCs. In this context, we conceive a syndrome-based block decoding approach for the classical Turbo Trellis Coded Modulation (TTCM), whose performance is investigated for transmission over an Additive White Gaussian Noise (AWGN) channel as well as over an uncorrelated Rayleigh fading channel.

Pursuing our objective of designing efficient QECCs, we next consider the construction of Hashing-bound-approaching concatenated quantum codes. In this quest, we appropriately adapt the conventional non-binary EXIT charts for Quantum Turbo Codes (QTCs) by exploiting the intrinsic quantum-to-classical isomorphism. We further demonstrate the explicit benefit of our EXIT-chart technique for achieving a Hashing-bound-approaching code design. We also propose a generically applicable structure for Quantum Irregular Convolutional Codes (QIRCCs), which can be dynamically adapted to a specific application scenario with the aid of the EXIT charts. More explicitly, we provide a detailed design example by constructing a 10-subcode QIRCC and use it as an outer code in a concatenated quantum code structure for evaluating its performance.

Working further in the direction of iterative code structures, we survey Quantum Low Density Parity Check (QLPDC) codes from the perspective of code design as well as in terms of their decoding algorithms. Furthermore, we propose a radically new class of high-rate row-circulant Quasi-Cyclic QLDPC (QC-QLDPC) codes, which can be constructed from arbitrary row-circulant classical QC-LDPC matrices. We also conceive a modified non-binary decoding algorithm for homogeneous Calderbank-Shor-Steane (CSS)-type QLDPC codes, which is capable of alleviating the problems imposed by the unavoidable length-4 cycles. Our modified decoder outperforms the state-of-the-art decoders in terms of their Word Error Rate (WER) performance, despite imposing a reduced decoding complexity. Finally, we intricately amalgamate our modified decoder with the classic Uniformly-ReWeighted Belief Propagation (URW-BP) for the sake of achieving further performance improvement.

Declaration of Authorship

I, Zunaira Babar, declare that the thesis entitled *Quantum Error Correction Codes* and the work presented in the thesis are both my own, and have been generated by me as the result of my own original research. I confirm that:

- this work was done wholly or mainly while in candidature for a research degree at this University;
- where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated;
- where I have consulted the published work of others, this is always clearly attributed;
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- parts of this work have been published as: [1, 2, 3, 4, 5, 6, 7]

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List of Publications

Journal Papers:

1. **Zunaira Babar**, Panagiotis Botsinis, Dimitrios Alanis, Soon Xin Ng and Lajos Hanzo, "Fifteen Years of Quantum LDPC Coding and Improved Decoding Strategies", *IEEE Access (to be submitted).*

2. **Zunaira Babar**, Panagiotis Botsinis, Dimitrios Alanis, Soon Xin Ng and Lajos Hanzo, "Construction of Quantum LDPC Codes from Classical Row-Circulant QC-LDPCs", *IEEE Communications Letters (to be submitted).*

3. **Zunaira Babar**, Panagiotis Botsinis, Dimitrios Alanis, Soon Xin Ng and Lajos Hanzo, "The Road from Classical to Quantum Codes: a Hashing Bound Approaching Design Procedure", *IEEE Access*, vol.3, pp. 146-176, Mar. 2015.

4. **Zunaira Babar**, Soon Xin Ng and Lajos Hanzo, "EXIT-Chart Aided Near-Capacity Quantum Turbo Code Design", *IEEE Transactions on Vehicular Technology*, vol.64, no.3, pp. 866-875, Mar. 2015.

5. **Zunaira Babar**, Soon Xin Ng and Lajos Hanzo, "Near-Capacity Code Design for Entanglement-Assisted Classical Communication over Quantum Depolarizing Channels", *IEEE Transactions on Communications*, vol. 61, no. 12, pp. 4801-4807, Dec. 2013.

6. **Zunaira Babar**, Soon Xin Ng and Lajos Hanzo, "Reduced-Complexity Syndrome-Based TTCM Decoding", *IEEE Communications Letters*, vol. 17, no. 6, pp. 1220-1223, Jun. 2013.

7. Panagiotis Botsinis, Dimitrios Alanis, **Zunaira Babar**, Soon Xin Ng and Lajos Hanzo, "Non-Coherent Quantum Multiple Symbol Differential Detection for Wireless Systems", *IEEE Access*, vol.3, pp. 569-598, May 2015.

8. Mark Wilde, Min-Hsiu Hsieh and **Zunaira Babar**, "Entanglement-Assisted Quantum Turbo Codes", *IEEE Transactions on Information Theory*, vol. 60, no. 2, pp. 1203-1222, Feb. 2014.

9. Panagiotis Botsinis, Dimitrios Alanis, **Zunaira Babar**, Soon Xin Ng and Lajos Hanzo, "Iterative Quantum-Assisted Multi-User Detection for Multi-Carrier Interleave Division Multiple Access Systems", *IEEE Transactions on Communications (under review).*

10. Abdulah Jeza Aljohani, **Zunaira Babar**, Soon Xin Ng, and Lajos Hanzo, "Distributed Source-Channel Coding using Reduced-Complexity Syndrome-Based TTCM", *IEEE Communications Letters (to be submitted).*

11. Dimitrios Alanis, Panagiotis Botsinis, **Zunaira Babar**, Soon Xin Ng and Lajos Hanzo, "Non-Dominated Quantum Iterative Routing Optimization for Wireless Multihop Networks", *IEEE Access (to be submitted).*

Conference Papers:

1. **Zunaira Babar**, Soon Xin Ng and Lajos Hanzo, "EXIT-Chart Aided Code Design for Symbol-Based Entanglement-Assisted Classical Communication over Quantum Channels", *IEEE Vehicular Technology Conference (VTC-Fall)*, Sep. 2014, Vancouver, BC.

2. Hung Nguyen, **Zunaira Babar**, Soon Xin Ng, Matteo Mazzotti, Lorenzo Iacobelli and Lajos Hanzo, "Network Coded MIMO Aided Cooperative Communications in the Ambulance-and-Emergency Area", *International Conference on Selected Topics in Mobile and Wireless Networking (MoWNet)*, Sep. 2014, Rome, IT.

Poster Presentation:

1. **Zunaira Babar**, Panagiotis Botsinis, Dimitrios Alanis, Soon Xin Ng and Lajos Hanzo, "Quantum-Aided Solutions in Wireless Systems", *International Workshop on Quantum Communication Networks*, Jan. 2014, Leeds, UK.

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