Strategic Argumentation

Wioletta Dziuda[1]
Kellogg School of Management
Managerial Economics and Decision Sciences
wdziuda@kellogg.northwestern.edu

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Abstract

I analyze a disclosure game between an uninformed decision maker and an informed but possibly biased expert. The relevant information is contained in a set of arguments. The expert can disclose each argument credibly, but he cannot prove whether he has disclosed everything.

In all equilibria some, but not all, information is revealed. The biased expert exaggerates his reports in favor of his preference, yet he does not suppress all of the unfavorable information. The decision maker takes balanced reports at face value, but is skeptical about the unbalanced ones. In the latter case, she chooses the alternative favored by the biased expert only if the expert can provide her with enough favorable arguments.

The decision maker is better-off when she is familiar with the choice problem, and she is more likely to be persuaded in complex situations.

Keywords Strategic communication, persuasion, argumentation, expert, disclosure games

1 Introduction

Consider a first-time camera buyer who wants to compare all relevant characteristics of different cameras. As an inexperienced camera buyer, she may be uninformed about the complexity of the product; that is, she may not know which and how many technical specifications are important for taking quality pictures. The salesman knows all the relevant features and can credibly disclose each of them to the buyer. Disclosed information is credible because of different reasons: the salesperson may be able to prove the size of the display and zoom by demonstrating them, the consumer may be able to test the features by using the camera, or liability laws may make it unprofitable for the salesperson to lie about any feature. One does not, however, expect the salesperson to always disclose all information to the buyer. In addition to revealing favorable characteristics of the camera, he may also mention some unfavorable ones—for example, he may mention its short battery life—but he will probably conceal some unfavorable features. This observation is puzzling in light of the unravelling result, which states that when the expert cannot lie, he reveals all information even if the parties have conflicting interests.

There are many situations that exhibit similar features: an investor consulting a financial adviser before choosing between two investment options, a patient listening to a doctor before selecting a treatment, or a reader of an academic or newspaper article before forming opinion. This paper shows that unravelling may fail in such situations and identifies the reasons for this failure. It also shows that although information favorable to the expert is always disclosed, some unfavorable information is likely to be disclosed as well. Finally, it identifies conditions under which there is more disclosure of information.

The model has the following structure. A decision maker consults an expert to help her choose between two alternatives: *Right* and *Left.* The state of nature is a set of random variables, called *arguments,* each of which favors one alternative. The quality of each alternative is a fraction of the arguments in its favor, and the decision maker prefers *Right* to *Left* only if its quality is sufficiently high. The number of arguments is itself a random variable, which is known to the expert but not to the decision maker. The expert observes all of the arguments and truthfully reports a subset of them to the decision maker before the decision maker makes a choice. To focus on robust equilibria I analyze a game in which the players' preferences are uncertain; both the threshold quality above which the decision maker chooses *Right* and the expert's preference are private information. The expert can be either an honest type who reveals all of the arguments, a persuader toward *Right*, or a persuader toward *Left.* A persuader wants the decision maker to choose the persuader's preferred alternative.

Full disclosure of information is not an equilibrium; if it were, the decision maker would take all reports at face value, but then the expert would have an incentive to reveal only favorable arguments. Unravelling fails because the expert is unable to prove to the decision maker whether he has disclosed all arguments: the decision maker does not know, and the expert cannot prove how many arguments there are.

I first analyze a simple version of the game in which the expert can be either honest or a persuader toward *Right.* In this game, aruments in favor of *Right* and states with many such arguments are favorable to the persuader. In any equilibrium, the persuader biases his reports in favor of his preference, but does not suppress unfavorable information altogether. The decision maker takes at face value all reports that have a high proportion of arguments unfavorable to the persuader. For all other reports, the decision maker bases her choice only on the number of favorable arguments, disregarding the unfavorable ones. The reason for this is as follows. The decision maker must be sufficiently skeptical about reports with a small number of arguments in favor of *Left* –that is, she must believe that many more arguments in favor of *Left* are likely to exist – because they are available to the persuader in unfavorable states. Moreover, she must be equally skeptical about all such reports; that is, she must hold the same low posterior belief about the quality of *Right.* Otherwise, the persuader would mislead her by choosing a report about which she is least skeptical.

This intuition raises doubts about whether any information can be transmitted when there are three types of experts: an honest expert, a persuader toward *Right,* and a persuader toward *Left.* In this setting, the decision maker does not know which alternative the expert favors; hence, she does not know whether she should be skeptical about the reported quality of *Right* or *Left.* In section 4 I analyze a game with three types of experts and show that a persuader can successfully transmit information only if he separates himself from the persuader of the other type, and he always chooses to do so, if possible. This is a surprising finding. Since the persuader benefits from pooling with the honest expert, we might expect him to benefit from pooling with the expert with the opposite bias. However, this turns out not to be true in every state. When the persuader has many arguments favorable to him, he benefits more from proving this than from pooling with the other type of persuader. Hence, upon hearing many arguments in favor of one alternative, the decision maker is able to infer toward which alternative the expert may be biased and then bases her choice solely on these arguments. However, when the persuader has few favorable arguments, he prefers to pool with the persuader of the other type and thus benefit from the decision maker's uncertainty about what type of arguments have been concealed. As a result, the decision maker ignores the expert's recommendation if he supports it with too few arguments.

The equilibrium use of unfavorable arguments is consistent with everyday experience. For example, a salesman may mention a car's long acceleration time, or an author may mentions findings that disagree with his agenda. Some commercials use two-sided messages – messages containing arguments both for and against a given alternative – for example, an advertisement attempting to persuade consumers of superiority of dBase IV software disclosed that it was more costly and worse at handling errors than competing products.[1] Psychological research has shown that two-sided messages are more persuasive and increase the perceived truthfulness of the expert.[2] The current model can be viewed as a game-theoretical foundation for these observations. In the model, two-sided messages are not more persuasive, but neither do they harm the expert, and in equilibrium the persuader must use them to avoid revealing his type. However, adding a small number of naive decision makers who take the expert's messages at face value makes the rational decision maker more likely to be persuaded by the expert using a two-sided message, as shown in section 3.2. In such a game, the persuader has an incentive to bias his reports to influence the naive decision makers, which is why a rational decision maker is more skeptical about reports with few unfavorable arguments.[3]

In this paper, the inability of the expert to prove whether he has disclosed all information stems from two assumptions: first, that there is no credible message con-

[1] See Pechmann (1992). Another example is the case of Continental Airlines acknowledging a variety of past problems such as canceled flights and lost luggage when trying to persuade the clients about its new commitment to quality (Crowley and Hoyer, 1994).

[2] See, for example, Smith and Hunt (1978) and Anderson and Golden (1984).

[3] Crowley and Hoyer (1994) discuss many psychological theories for why two-sided messages are more persuasive. The intuition in this paper about persuasiveness of two-sided messages is closest to attribution theory, which says that decision makers attribute claims either to the persuader or to the honest expert. Including unfavorable arguments enhances the credibility of the expert.

firming full disclosure; and second, that the decision maker is uncertain about how many arguments exist. Varying the prior distribution of arguments amounts to relaxing or strengthening the second assumption. In particular, if the prior distribution of arguments is degenerate, the expert proves that he has revealed all arguments simply by doing so. In Section 5, I show that the utility of the decision maker increases as the prior distribution of the arguments becomes less dispersed. As this distribution becomes less dispersed, the decision maker has a more precise estimate of how many arguments she should consider; hence, she can better estimate the number of arguments that the expert has concealed. The dispersion of the prior distribution of arguments may be interpreted in two ways. First, it may represent the complexity of the decision problem. For example, when reading a newspaper, the decision maker may find it difficult to estimate how many relevant facts exist for each particular problem. Second, the dispersion of the distribution of arguments may represent the degree of the decision maker's familiarity with the problem. Hence, this paper finds that less informed decision makers facing complex choice problems benefit most from mandatory disclosure. It is well known that in many situations it is impossible to ascertain whether the expert possesses undisclosed information; therefore, mandatory disclosure policies may be difficult to implement. More broadly, my finding suggests that in such situations it is important to have a policy of informing decision makers about the relevant dimensions of the decision problems, but not necessarily about the values these dimensions take. In terms of the motivating example, educating consumers about the relevant features of cameras in general will lead to more information disclosure.

Since Grossman (1981), Milgrom (1981), Milgrom and Roberts (1986), and Matthews and Postlewaite (1985) first laid out the unravelling argument, the research has focused on identifying situations in which this argument may fail.[4] Viscusi (1978) and Jovanovic (1982) show that the expert reveals only favorable states when disclosure is costly. In Fishman and Hagerty (2003), information transmission is hindered by the presence of decision makers who can verify the event of disclosure, but do not understand the disclosed information. Dye (1985), Shin (1994a), Shin (1994b), and Shin (2003) show that unravelling may fail if there is uncertainty about how well informed

[4] A parallel strand of literature focuses instead on information transmission when talk is cheap; that is, when the expert cannot prove anything, but can send any message at no cost. See, for example, Crawford and Sobel (1982), Krishna and Morgan (2001), and Battaglini (2002).

the expert is. In a dynamic setup, Grubb (2007) shows that unravelling may be hindered further if senders want to build a reputation for being uninformed. Although in the present paper disclosure is costless, all decision makers are fully rational, and the expert is fully informed, unravelling fails nevertheless.

The papers closest to mine are Shin (1994a) and Wolinsky (2003). However, the current paper differs from Shin (1994a) in two important ways. First, in Shin (1994a), the expert may have imprecise information about the state of nature and can reveal what he knows credibly, but he cannot prove that his information is imprecise. In contrast, in the current model the expert always knows the state of nature, but he cannot prove that he has disclosed all available arguments. However, even though these are different assumptions, the logic at work is somewhat similar: the expert is unable to prove whether he has disclosed everything. Second, Shin (1994a) focuses on the class of equilibria in which the expert reveals only favorable information. I allow for uncertainty about the preferences of the expert, with the result that the complete suppression of unfavorable information cannot be part of an equilibrium.[5]

Wolinsky (2003) introduces uncertainty about the preference of the expert in a game with fully informed experts. In his model, unravelling fails because the decision maker cannot hold a belief unfavorable to the expert, since she does not know the expert's preferences. In contrast, in my model uncertainty about the expert's preferences is not crucial to the failure of information revelation. In both papers the uncertainty about the expert's preferences results in the use of reports that would not be most favorable if taken at face value. In contrast to this paper, however, in Wolinsky (2003) the strategy of the expert cannot be interpreted as the use of two–sided messages.

The nature of disclosure suggests that the message space available to the expert is discrete, and this is what Shin (1994a), Shin (1994b), and Wolinsky (2003) assume. One technical contribution of this paper is to model the number of arguments as a continuous variable. This makes the model more tractable and allows for more generality.[6]

The paper is organized as follows. Section 2 describes the game. Section 3 analyzes a version of the model in which the expert is either honest or a persuader toward *Right.*

[5] For more on this difference, see Section 5.1.

[6] From the modeling perspective, Glazer and Rubinstein (2001) and (2004) also analyze a model in which information is a collection of arguments, but they focus on communication when there is boundary on the number of arguments that can be revealed.

Section 4 extends the analysis to the case in which the expert can be any of three types. Section 5 analyzes the impact of uncertainty that the decision maker faces on the equilibrium outcome. Section 6 provides a summary and conclusions.

2 The Model

The environment

There are two alternatives: *Right* and *Left.* A state of nature is a tuple $(L, R) \in R_+^2$. L represents the number of arguments in favor of *Left,* and R represents the number of arguments in favor of *Right.* Let $f(L, R)$ be the prior probability density function over the state space, and $F(L, R)$ be the corresponding distribution function. The distribution over the state space is common knowledge.[7]

There are two players: an expert and a decision maker.

The expert

The expert observes the state of nature (L, R) and sends a report to the decision maker. A report is a tuple (λ, ρ), where λ is the number of arguments in favor of *Left* and ρ is the number of arguments in favor of *Right* that the expert reveals. A report $(\lambda, \rho) \in R_+^2$ is *feasible* in state (L, R) if $\lambda \leq L$ and $\rho \leq R$, and in each state the expert can send any report from the feasible set at no cost. This implies that the expert can truthfully disclose any subset of the existing arguments, but cannot credibly convey that he has disclosed all of them.

There are three types of experts: an honest expert, H; a persuader toward *Right,* P_r; and a persuader toward *Left*, P_l. An honest expert is *non-strategic* and reveals all of the arguments. A persuader toward alternative A wants the decision maker to choose A, independent of the state of nature; that is, he maximizes $\Pr\{A \text{ is chosen}\}$. The probability that the expert is of type $i \in \{P_l, P_r, H\}$ is π_i. A strategy of a type i persuader, $m_i((\lambda, \rho) \mid (L, R))$, specifies for each (L, R) the probability distribution over the set of feasible reports. The expert is said to report *fully* if he reveals all of the arguments.

The decision maker

[7] Note that arguments are continuous variables. In my motivating examples, the number of arguments is a discrete variable, but modeling L and R as continuous variables makes the model significantly more tractable and allows it to obtain general results without putting much structure on $f(L, R)$. Section 3.3 discusses the limitations of this assumption.

Define the quality of a given alternative as the fraction of arguments in its favor, $q_R = \frac{R}{R+L}$ and $q_L = \frac{L}{R+L}$. The utility of the decision maker is:

$$U(Right|L,R) = q_R - \theta,\ U(Left|L,R) = q_L + \theta - 1 \tag{1}$$

where $\theta \in [0,1]$ is a preference parameter. In short, the decision maker chooses *Right* if and only if $E[q_R|\lambda,\rho] \geq \theta$.

The parameter θ describes an ex ante preference of the decision maker. For example, a consumer may have some intrinsic preference for Canon cameras over Panasonic cameras, a shareholder may prefer stocks of environmentally friendly companies, or a voter may prefer a Republican candidate because of family tradition, all other things being equal. Nature chooses θ according to a continuous probability density function $g(\theta)$ with full support, with the corresponding distribution function $G(\theta)$. The decision maker observes her θ, but the expert does not.

The game

The game proceeds as follows. First, nature determines the type of expert $i \in \{P_l, P_r, H\}$ and the set of arguments (L,R). The expert observes his type and the state of nature (L,R), and sends a report (λ,ρ) to the decision maker. The decision maker observes her type and the report, and chooses one of the alternatives.

Technical assumptions

Assume that $f(L,R)$ is continuous with full support on R_+^2. Let $f^L(L|R)$ denote the conditional density of L given R, and $f^R(R|L)$ the conditional density of R given L. Let $F^L(L|R)$ and $F^R(R|L)$ be the corresponding distribution functions. I impose the following regularity conditions:

$$\frac{dF^L(L|R)}{dR} \geq 0,\ \frac{dF^R(R|L)}{dL} \geq 0. \tag{2}$$

Intuitively, condition 2 says that the presence of an additional argument in favor of one alternative does not make the opposing arguments more likely. This rules out situations in which "good news is bad news," that is, in which a higher number of arguments favoring a given alternative makes this alternative less likely to be attractive.

Graphic representation

The triangle in Figure 1 represents the state space and the report space. Define

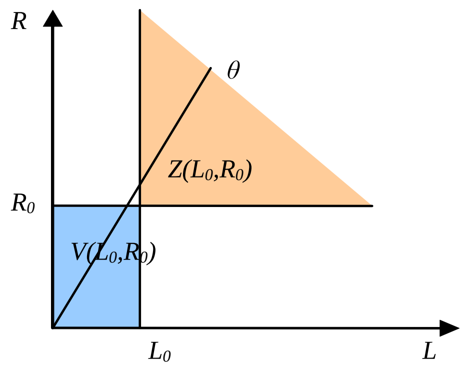


Figure 1: This figure represents the state space. $V(L_0, R_0)$ is the set of feasible reports for (L_0, R_0) and $Z(L_0, R_0)$ is the set of states of nature in which report (L_0, R_0) is feasible.

$Z(\lambda, \rho) = \{(L, R) \in S : L \geq \lambda \text{ and } R \geq \rho\}$ and let $V(L, R)$ be the set of feasible reports in state (L, R). If the state of nature is (L_0, R_0), then the shaded region $V(L_0, \rho_0)$ is the set of feasible reports. The shaded region $Z(L_0, R_0)$ is the set of states of nature that allow the expert to send a report (L_0, R_0).

The line θ represents the states of nature that generate the same quality, $q_R = \theta$. The decision maker of type θ prefers to choose *Right* if the state of nature lies above this line but *Left* otherwise.

Equilibrium concept

I look for a perfect Bayesian equilibrium of this game. The decision maker in this game has a very limited role: she chooses *Right* if she believes that the quality of *Right* is above θ. Given this, from the perspective of the expert the probability that the decision maker chooses *Right* is a strictly increasing function of the belief. Therefore, one can transform this game into a game with one player only, in which the persuader toward A maximizes the belief of the decision maker about q_A, where the belief is formed using Bayes' rule.[8]

Let $\eta(\lambda, \rho)$ be the equilibrium belief of the decision maker about q_R if she observes report (λ, ρ). A perfect Bayesian equilibrium is characterized by m_i for $i \in \{P_l, P_r\}$ and $\eta(\lambda, \rho)$ such that

[8] The honest expert uses all possible reports; therefore, there are no off-equilibrium beliefs.

1. m_i satisfies

$$\int_0^R \int_0^L m_i \left((\lambda, \rho) \,|\, (L, R) \right) d\lambda d\rho = 1 \text{ for all } (L, R) \in R_+^2;$$

2. if (λ^*, ρ^*) is in the support of $m_{P_r}\left(\cdot | \, (L, R)\right)$, then (λ^*, ρ^*) solves

$$\max_{(\lambda,\rho)\in V(L,R)} \eta\left(\lambda, \rho\right);$$

   and if (λ^*, ρ^*) is in the support of $m_{P_l}\left(\cdot | \, (L, R)\right)$, then (λ^*, ρ^*) solves

$$\min_{(\lambda,\rho)\in V(L,R)} \eta\left(\lambda, \rho\right);$$

3. $\eta\left(\lambda, \rho\right)$ is derived using Bayes' rule.

The above definition entails the immediate conclusion that the set of equilibria does not depend on the distribution of θ as long as it has full support.

3 One-sided Bias

In this section, I consider a situation in which the decision maker knows the direction of the potential bias of the expert. Let π be the probability that the expert is biased toward *Right,* i.e., is of type P_r; and $1-\pi$ be the probability that the expert is honest, i.e., is of type H.

The decision maker often knows which alternative the expert may favor. A sales representative may advise the customer honestly about the quality of his product, but he is certainly not interested in increasing the sales of competing products. A firm may report honestly, but if it does not, it is interested in maximizing its perceived value.

3.1 The properties of the equilibria

It is easy to see that there is no equilibrium with full information disclosure. If there were one, the expert's reports would be taken at face value, $\eta\left(\lambda, \rho\right) = \frac{\rho}{\lambda+\rho}$, and the persuader would prefer to conceal all unfavorable arguments, convincing the decision maker to choose *Right.* Proposition 1 states the less obvious result.

Proposition 1 *There is no babbling equilibrium.*

Proof All proofs are in the Appendix. ■

The intuition for Proposition 1 is as follows. In a babbling equilibrium all reports generate the prior belief $E\left[\frac{R}{R+L}\right]$. Consider a decision maker who receives many arguments in favor of *Right* (ρ_0 large) and few in favor of *Left* (λ_0 small). There is some probability that these reports come from the honest expert, and in such a case the quality of *Right* is close to 1. Hence, if the decision maker forms $E\left[\frac{R}{R+L}\right] < 1$, it must be that the persuader sends this report when the quality of *Right* is low, which is when there are many more arguments in favor of *Left* than in the report. But given condition 2, such a state is very unlikely relative to state (λ_0, ρ_0). Hence for ρ_0 large enough the decision maker believes it is very likely that the report came from the honest expert; her belief therefore must be close to 1.

Although there is no babbling equilibrium, there are many partially revealing equilibria in this game, which is not very surprising given that the message space is larger than the payoff-relevant space. However, all equilibria share properties described in Proposition 2.

Proposition 2 *In each equilibrium, for all ρ, there exists λ_ρ such that $\eta(\lambda, \rho)$ is constant for all $\lambda \in [0, \lambda_\rho)$. The persuader is indifferent among revealing any small number of unfavorable arguments and does not always suppress them completely. The belief $\eta(0, \rho)$ is weakly increasing in ρ.*

In all equilibria the persuader does not completely suppress the unfavorable information: his strategy includes revealing arguments that appear to oppose his interest. However, two-sided messages are not more persuasive. If the number of arguments in favor of *Left* that the expert reveals is low enough, the decision maker bases her decision solely on the number of arguments in favor of *Right:* $\eta(\lambda, \rho)$ is independent of λ.

The intuition for Proposition 2 is as follows. Assume that there exists ρ_0 such that the belief about the quality of *Right* when only favorable arguments are revealed is different from the belief when some unfavorable arguments are revealed; that is, $\eta(\lambda_0, \rho_0) \neq \eta(0, \rho_0)$ for an arbitrarily small λ_0. If the belief is higher when there are few unfavorable arguments than when there are none, $\eta(\lambda_0, \rho_0) > \eta(0, \rho_0)$, the persuader always tries to reveal a small number of unfavorable arguments. Hence, upon hearing a report with favorable arguments only, the decision maker must believe that the quality of *Right* is 1, which contradicts the assumption that including a small number of unfavorable arguments is beneficial. If the belief is lower when there are few unfavorable arguments than when there are none, $\eta(\lambda_0, \rho_0) < \eta(0, \rho_0)$, then the persuader suppresses the unfavorable arguments completely, and the decision maker must take the report with few unfavorable arguments at face value. In this case, however, the expert benefits from revealing a few unfavorable arguments. Even though he would be admitting that *Right* is not perfect, he would gain because his report would be perceived as credible.

Intuitively, the decision maker should be skeptical about reports with very few unfavorable arguments because the persuader can send them when the quality of *Right* is not very high. Moreover, if she is more skeptical about one report than another, the persuader would only use the latter, and her skepticism would not be rational. Hence, she forms the same belief upon seeing any small number of unfavorable arguments. This means that revealing a small number of unfavorable arguments does not hurt the persuader, and in equilibrium he uses these arguments in order to keep the decision maker's beliefs justified.

Given the multiplicity of equilibria, it is natural to ask whether some equilibria are more plausible than others. In the remainder of the paper I focus on equilibria in which the belief function $\eta(\lambda, \rho)$ is continuous in reports. I do this in part because Proposition 2 suggests that focusing on one type of equilibria is without much loss of generality. Nevertheless, I justify my choice in section 3.3 further.

Proposition 3 *There is a unique equilibrium belief function* $\eta(\lambda, \rho)$ *that is continuous in reports* (λ, ρ)*. All equilibria characterized by this function are outcome-equivalent. In any such equilibrium,* $\eta(\lambda, \rho)$ *is strictly increasing in* ρ*, and for each* ρ*, there exists* $\lambda_\rho > 0$*, defined by*

$$\frac{\rho}{\rho + \lambda_\rho} = \Pr(H|\lambda \leq \lambda_\rho, \rho) E[q_R|L \leq \lambda_\rho, R = \rho] + \Pr(P_r|\lambda \leq \lambda_\rho, \rho) E[q_R|R = \rho] \tag{3}$$

such that

i. P_r *reveals all arguments in favor of* Right*,* $\rho = R$ *for each* R*;*

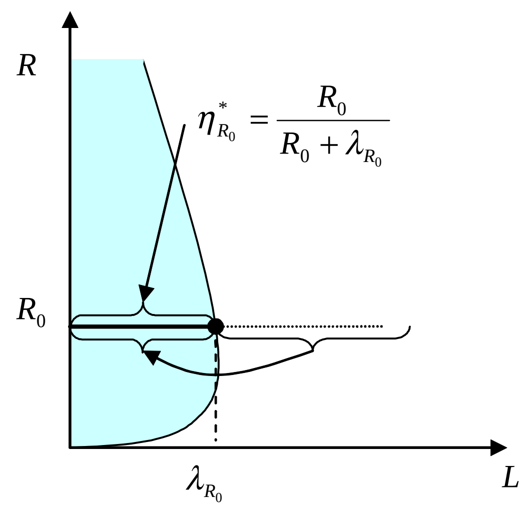


Figure 2: The details of the continuous equilibria. The shaded region represents the ambiguity area.

ii. P_r *reveals a subset of arguments in favor of* Left, $\lambda \leq \min\{L, \lambda_\rho\}$, *using a strategy that results in* $\eta(\lambda, \rho) = \frac{\rho}{\rho+\lambda_\rho} \equiv \eta^*_\rho$ *for all* $\lambda \leq \lambda_\rho$.

Proposition 3 says that there is a unique equilibrium belief function that is continuous in reports. Given that the persuader always tries to induce the highest possible belief, this implies that all continuous equilibria are outcome-equivalent. Figure 2 represents all of these equilibria for a fixed π and $f(L, R)$.[9] The white area, which I call the *revealing area*, is the set of reports that are used in equilibrium only by H. The shaded region, which I call the *ambiguity area,* is the set of reports used in equilibrium also by P_r. Hence, the ambiguity area includes all reports that do not allow the decision maker to identify the type of expert. The boundary of the ambiguity area is determined by $\lambda_{\rho=R}$ defined by equation (3).

In any continuous equilibrium, the persuader reveals all of the arguments that favor *Right* and some arguments that favor *Left.* The highest number of arguments in favor of *Left* that the persuader reveals for any R is λ_R. After observing a report from the revealing area, the decision maker knows that the expert has reported fully, and she takes this report at face value, that is, $\eta(\lambda, \rho) = \frac{\rho}{\rho+\lambda}$. After observing a report from the ambiguity area, she forms her belief based only on $\rho : \eta(\lambda, \rho) = \frac{\rho}{\rho+\lambda_\rho}$. Given

[9] The comparative statics is in Section 5.

this belief function, the persuader is indifferent among sending any report (λ, ρ) such that $\rho = R$ and $\lambda \leq \lambda_R$. However, in equilibrium he must use a strategy that supports the decision maker's beliefs.

The persuader can use many strategies that support the continuous belief function, but they all must generate a belief that is constant in λ for $(\lambda, \rho) : \lambda \leq \lambda_\rho$. This belief is characterized by

$$\eta(\lambda, \rho) = \Pr(H|\lambda, \rho) \frac{\rho}{\rho + \lambda} + (1 - \Pr(H|\lambda, \rho)) E\left[\frac{\rho}{\rho + L}|\lambda, \rho, P_r\right]. \tag{4}$$

If the expert is honest, then a smaller number of arguments in favor of *Left* implies a higher quality of *Right.* The strategy of the persuader must offset this effect. Differentiating both sides of equation 4 with respect to λ, we get

$$\begin{aligned} & \frac{d\Pr(H|\lambda, \rho)}{d\lambda}\left(\frac{\rho}{\rho + \lambda} - E\left[\frac{\rho}{\rho + L}|\lambda, \rho, P_r\right]\right) \\ = \; & \Pr(H|\lambda, \rho) \frac{\rho}{(\rho + \lambda)^2} - (1 - \Pr(H|\lambda, \rho)) \frac{dE\left[\frac{\rho}{\rho + L}|\lambda, \rho, P_r\right]}{d\lambda}. \end{aligned}$$

The expression in brackets on the left-hand side is positive because the persuader sends λ only if $L \geq \lambda$. This implies that in equilibrium either $\frac{d\Pr(H|\lambda,\rho)}{d\lambda} > 0$ or $\frac{dE\left[\frac{\rho}{\rho+L}|\lambda,\rho,P_r\right]}{d\lambda} > 0$. This means that as λ increases, either the posterior belief that the expert is honest increases, or the expected quality of *Right* conditional on the expert being a persuader increases – or both. The first effect captures the intuition that two-sided arguments are more credible (although not more persuasive). The second effect is less intuitive; it says that the less favorable the state is, the more likely the persuader is to send more extreme reports . Although there is no guarantee that revealing unfavorable information increases the credibility of the expert in any particular equilibrium, there always exist equilibria in which this is true. In one such equilibrium the persuader reports fully if $L \leq \lambda_\rho$; otherwise, he randomizes over $\lambda \in [0, \lambda_\rho]$ using some probability density function $s(\lambda)$. For $s(\lambda)$ to be an equilibrium, it must be decreasing in λ, which means that the persuader randomizes over how many arguments to reveal, but is more likely to reveal fewer of them. Under this strategy revealing an additional argument in favor of *Left* increases the credibility of the expert, $\Pr(H|\lambda, \rho)$, but decreases the estimate of the quality of

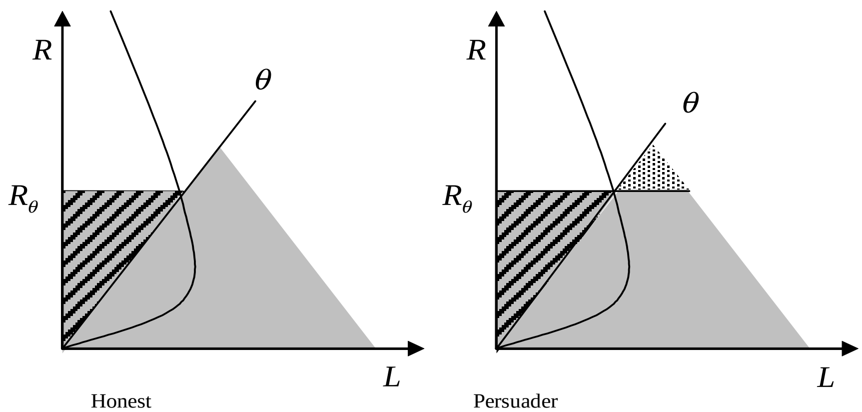


Figure 3: The behavior of the decision maker. The triangles represent the choice of the decision maker given the state of nature and given that the expert happens to be an honest type or a persuader, respectively.

Right conditional on the expert being honest.

Figure 3 shows the behavior of the decision maker of type θ in any continuous equilibrium. The first triangle represents her decision as a function of state if she happens to face the honest expert, and the second triangle represents her decision if she happens to face the persuader. The decision maker prefers *Right* in the states that lie above the θ line and prefers *Left* otherwise. If she observes a report from the revealing area, she chooses *Right* if the report lies above the θ line. If she observes a report from the ambiguity area, R_θ is the number of arguments for *Right* for which $\eta^*_{R_\theta} = \theta$. This means that when seeing a report from the ambiguity area, the decision maker chooses *Right* if $R \geq R_\theta$ and *Left* if $R < R_\theta$.

In Figure 3 the shaded areas represent the states in which the decision maker chooses *Left.* When the number of arguments that favor *Right* is low enough, the decision maker chooses *Left* even if she receives an extreme report. The striped area represents the states in which *Right* is optimal but *Left* is chosen. Note that in the states from the striped area, every player of this game prefers the decision maker to choose *Right,* but she nevertheless chooses *Left* in equilibrium. The dotted area represents *successful persuasion;* that is, the states in which *Left* is actually better, but the decision maker is persuaded to choose *Right*.

The inability of the expert to prove whether he has disclosed all relevant information hinders unravelling and causes the decision maker to make suboptimal choices even in situations in which all players of the game agree on the choice. The decision maker is persuaded to select *Right* in complex situations in which the persuader has many favorable arguments at his disposal. However, she fails to choose *Right* in simple situations. In this model mandatory disclosure would clearly improve the welfare of the decision maker.

3.2 Naive decision makers

One prediction of this model is that experts may use arguments that seemingly go against their interests. This is consistent with casual observations and psychological research. However, this paper does not explain why people may be more likely to be convinced by two-sided messages. In my model in any equilibrium, if the decision maker cannot identify the type of expert, she bases her decision solely on the number of arguments in favor of *Right,* disregarding completely the arguments in favor of *Left*. In this section, I show that if the decision maker might be naive, that is, if there is a chance that she would take the expert's recommendation at face value, a rational decision maker is more likely to be persuaded by two-sided reports.[10]

Assume that with probability $\mu < 1$ the decision maker is naive and believes that $(L, R) = (\lambda, \rho)$. As before, $\eta(\lambda, \rho)$ denotes the belief of a rational decision maker. The persuader maximizes the probability that *Right* is chosen, which is now

$$\begin{aligned} \Pr(\textit{Right} \text{ is chosen}|\lambda, \rho) &= \mu \Pr\left(\theta \leq \frac{\rho}{\rho + \lambda}\right) + (1 - \mu) \Pr(\theta \leq \eta(\lambda, \rho)) \\ &= \mu G\left(\frac{\rho}{\rho + \lambda}\right) + (1 - \mu) G(\eta(\lambda, \rho)), \qquad (5) \end{aligned}$$

where $G(\theta)$ is the c.d.f. of θ. Define $p(\lambda, \rho) \equiv \Pr(\textit{Right} \text{ is chosen}|\lambda, \rho)$. Proposition 4 says that in any equilibrium $p(\lambda, \rho)$ has the same properties as $\eta(\lambda, \rho)$ in the model without naive decision makers, namely, $p(\lambda, \rho)$ cannot depend on λ for small λs.

Proposition 4 *In any equilibrium, for all ρ, there exists λ_ρ such that $p(\lambda, \rho)$ is constant for all $\lambda < \lambda_\rho$. For each ρ and $\lambda < \lambda_\rho$, the belief of the rational decision*

[10] For the impact of introducing naive decision makers into cheap talk games, see, for example, Chen (2006).

maker $\eta(\rho,\lambda)$ is continuous and strictly increasing in λ, and satisfies:

$$g(\eta(\lambda,\rho))\frac{d\eta(\lambda,\rho)}{d\lambda}=\frac{\mu}{1-\mu}g\left(\frac{\rho}{\rho+\lambda}\right)\frac{\rho}{(\rho+\lambda)^2}.$$

The model with naive decision makers explains why two-sided messages may be more persuasive. The expert faces a trade-off: revealing an additional unfavorable argument decreases his chances of convincing a naive decision maker, but increases his credibility and probability of success if the decision maker is rational.

In the sections that follow, in order to simplify the present analysis, I will assume that all decision makers are rational. However, later I discuss the effect of adding naive decision makers to the model with three types of expert.

3.3 Robustness

3.3.1 Selection of continuous equilibria

Among all equilibria, the set of equilibria with the continuous belief function stands out. All continuous equilibria are outcome-equivalent, that is, the decision maker makes identical choices in all continuous equilibria. The same is not true of the set of discontinuous equilibria.

Proposition 5 establishes another attractive feature of continuous equilibria.

Proposition 5 *The ex-ante utility of the decision maker is the highest in continuous equilibria.*

In continuous equilibria, the persuader induces a different belief for each R, while this is not true in discontinuous equilibria. We can find $R_1 > R_2$ such that some types of the decision maker, when facing the persuader, choose *Right* when $R = R_1$ and *Left* when $R = R_2$ in a continuous equilibrium, but choose the same alternative in both cases in some discontinuous equilibrium. This suggests that the decision maker is worse-off in this discontinuous equilibrium if she faces the persuader. She may be better-off when she faces the honest expert if the ambiguity area is smaller, in which case the honest expert can signal higher qualities of *Right* than he could in continuous equilibria. It turns out that the loss due to worse decision making when facing the persuader always outweighs the benefit from better decision making when facing the honest expert. The reason for this is that the affected decision maker has θ high

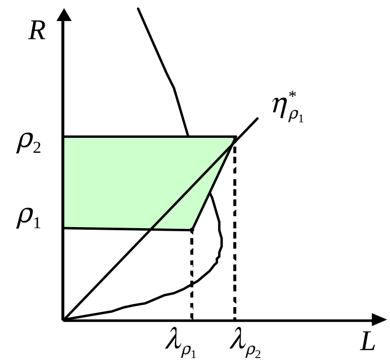


Figure 4: An example of a discontinuous equilibrium.

enough that the probability of facing the honest expert and a state being $q_R > \theta$ – that is, the probability of improving the decision – is relatively low.

In the rest of the paper I focus on continuous equilibria only.[11] However, I want to shed more light on what discontinuous equilibria look like. Figure 4 shows a representative equilibrium (I derive more features shared by all equilibria in the proof of Proposition 2). In this equilibrium the belief induced when only few arguments in favor of *Left* are revealed is strictly increasing for $\rho \in (0, \rho_1)$ and $\rho \in (\rho_2, \infty)$ and is constant for $\rho \in (\rho_1, \rho_2)$. The curve represents λ_ρ for $\rho \in (0, \rho_1)$ and $\rho \in (\rho_2, \infty)$. The shaded trapezoid represents reports that generate the same belief $\eta^*_{\rho_1}$. Additionally, all reports lying in the rectangle that completes the trapezoid generate either the same belief as the trapezoid, or are sent only by H.

3.3.2 Utility function

In this model the decision maker cares only about the quality of each alternative; the total number of arguments does not enter her utility function. This reflects the idea that the set of all arguments carries *perfect* information about the state of nature, and the number of arguments measures only the complexity of the problem. For example, computers have more features than memory cards, and buying a house

[11] The previous version of this paper contained a proposition that if we perturb the game by adding a small fixed cost of concealing information, its unique equilibrium converges to a continuous equilibrium. Results upon request.

requires considering more aspects than buying a car. The decision maker is uncertain about this complexity, but the complexity itself does not affect her preferences. There is no reason why consumers should require stronger proof when buying a more complex product.

The qualitative results of this paper still hold if the decision maker's utility depends on the quality of the chosen alternative, but the expected value of the quality depends on the total number of arguments as well as their proportion. In equilibrium, the expected quality given the expert's report must be constant for some λ; otherwise the expert would use only the reports generating the highest expectation. More generally, for any utility specification the probability of *Right* being chosen must be constant for any small number of unfavorable arguments. Hence, in all these models the decision maker bases her decision only on the number of favorable arguments, and the expert must use two-sided messages.

3.3.3 Continuity of arguments

The intuition for Proposition 2 makes it clear that the assumption of the continuity of arguments is not completely innocuous. If arguments are discrete, revealing one unfavorable argument proves that the quality of *Right* is significantly lower than 1, and this may outweigh the benefit from gaining credibility. However, complete suppression of unfavorable arguments may be a part of an equilibrium only if the probability of a state with no arguments in favor of *Left* is relatively high compared to the probability of the expert being a persuader and having at least one such argument. If the distribution of arguments is not very skewed toward states with arguments only in favor of *Right*, and the probability that the expert is a persuader is high enough, then the main conclusion of Proposition 2 would hold in a discrete model.

3.3.4 Benevolent expert

In this model the honest expert reveals all of the arguments. Alternatively, the honest expert may want to maximize the utility of the decision maker, i.e., he may be benevolent. One can easily see, however, that any continuous equilibrium of the original game is still an equilibrium of a game with a benevolent type of expert, and the benevolent expert behaves like an honest expert. To see this, note that if the state of nature lies in the revealing area, the benevolent expert cannot do better than to report fully, because in this way he induces the correct belief. If the state of nature lies in the ambiguity area, however, the benevolent expert would like to induce a higher belief than the one induced in equilibrium, but there is no feasible report that can achieve this; therefore, full reporting is once again optimal.

When the expert is benevolent, however, there are more equilibria. In particular, it is no longer true that all reports must be used. As a result, by appropriately choosing the off-equilibria beliefs, one can support many implausible equilibria. Hence, assuming that the expert is honest is similar to (but not as strong as) using a refinement which requires off-equilibrium reports to be taken at face value.

4 Two-sided Bias

In this section, I consider a situation in which the expert can be biased toward either alternative. The expert can be P_r, P_l, or H with probabilities π_r, π_l, and π_H, respectively.

Sometimes the decision maker is uncertain not only about whether the expert is honest, but also about the potential bias of the persuader. A salesman may give honest advice, but he may have an interest in selling one particular product, and the decision maker may not know which product that is. Similarly, a scientist publishing a comparison of the performance of two drugs may be honest or biased, and the reader may not know which pharmaceutical company funded the research.

The previous section shows that when the expert is either honest or biased toward *Right*, the decision maker knows that all arguments in favor of *Right* have been revealed and uses these arguments to form her beliefs. Unless the expert reveals himself to be honest, she completely disregards the arguments that favor *Left.* When the expert can be biased in either direction, the decision maker cannot use the same logic; therefore, we can expect that much less information will be revealed. This is, however, only partially true. Proposition 6 describes the unique continuous equilibrium outcome. The persuader toward *Right* and the persuader toward *Left* separate themselves if they happen to receive many arguments in favor of their preferred alternatives. In these states, the decision maker can use the same skeptical approach to infer information as in the one-sided case . If the persuaders receive very few favorable arguments, however, little information is revealed.

Proposition 6 *There is a unique equilibrium belief function that is continuous. Any continuous equilibrium is characterized by the same parameters $\bar{R}, \bar{L}$ and by the functions $\lambda_\rho, \rho_\lambda$, such that*

i. *For all (L, R) such that $R \geq \bar{R}$, P_r reveals all arguments that favor* Right, *and reveals a subset of arguments that favor* Left: $\lambda \leq \min\{L, \lambda_\rho\}$.

ii. *For all (L, R) such that $L \geq \bar{L}$, P_l reveals all arguments that favor* Left, *and reveals a subset of arguments that favor* Right: $\rho \leq \min\{R, \rho_\lambda\}$.

iii. *There exists a double ambiguity area such that when $R < \bar{R}$, P_r sends reports from this area only; and when $L < \bar{L}$, P_l sends reports from this area only. The belief is constant for all reports in the double ambiguity area.*

iv. *λ_ρ and ρ_λ solve the following equations:*

$$\eta^*_\rho \equiv \frac{\rho}{\rho + \lambda_\rho} = \Pr(H|\rho, \lambda \leq \lambda_\rho) E[q_R|R = \rho, L \leq \lambda_\rho] + \Pr(P_r|\rho, \lambda \leq \lambda_\rho) E[q_R|R = \rho]; \tag{6}$$

$$\eta^*_\lambda \equiv \frac{\rho_\lambda}{\rho_\lambda + \lambda} = \Pr(H|\lambda, \rho \leq \rho_\lambda) E[q_R|L = \lambda, R \leq \rho_\lambda] + \Pr(P_l|\lambda, \rho \leq \rho_\lambda) E[q_R|L = \lambda]. \tag{7}$$

Figure 5 represents a continuous equilibrium for symmetric $f(L, R)$ and for $\pi_l = \pi_r$. In this equilibrium $\bar{R} = \bar{L}$ and $\eta(\bar{L}, \bar{R}) = \frac{1}{2}$. The striped area represents the ambiguity area for P_r, and the dotted area represents the ambiguity area for P_l. The ambiguity area for P_r contains all reports used by P_r and H only, while the ambiguity area for P_l contains all reports used by P_l and H only. The shaded square represents the set of reports that are used in equilibrium by all three types of expert, the *double ambiguity area.*

As before, each type of persuader biases his reports; reports that consist of many relatively balanced arguments are therefore sent only by the honest expert. Because each persuader biases the reports toward his preferred alternative, only H and P_r reveal many arguments in favor of *Right,* and only H and P_l reveal many arguments in favor of *Left.* Hence, reports that consist of many arguments in favor of only one side reveal the potential bias of the expert. After observing many arguments in

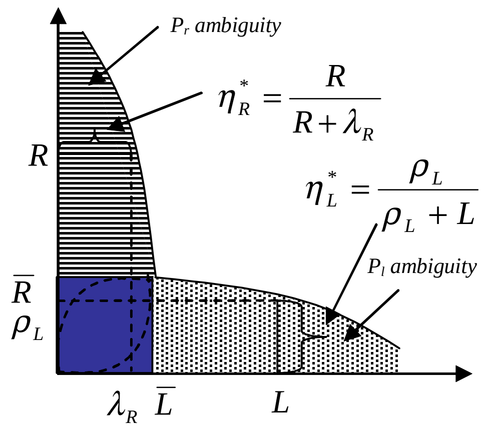


Figure 5: Two-sided bias.

favor of *Right* (reports from the P_r ambiguity area), the decision maker knows that she does not face P_l, and therefore also knows that all arguments in favor of *Right* have been revealed to her. Similarly, when she observes many arguments in favor of *Left* (reports from the P_l ambiguity area), she knows that she does not face P_r, and therefore also knows that all arguments in favor of *Left* have been revealed to her.

When the expert supports his recommendation with just few arguments, the decision maker ignores what the expert says. The intuition behind this finding is as follows. If many arguments are revealed, the decision maker expects that the expert has hidden only few arguments. So even if the expert reveals himself to be biased toward *Right*, if he reveals many supporting arguments, the decision maker is still likely to choose *Right*. On the other hand, if the expert reveals few arguments in favor of *Right*, it is more likely that he has concealed a lot of arguments in favor of *Left*. Hence, if the expert reveals himself to be biased toward *Right*, the decision maker would rather choose *Left*. This is why the experts have no incentive to separate themselves when they have few favorable arguments. Therefore, for small ρ and λ the equilibrium resembles a pure babbling equilibrium.

In the example shown in Figure 5 when the decision maker sees a report from the double ambiguity area, she chooses the alternative supported by her prior belief. However, this need not to be true if the case is not symmetric. For example, if $\pi_l \neq \pi_r$, then $\eta\left(\bar{L}, \bar{R}\right) \neq \frac{1}{2}$, which implies that hearing reports from the double ambiguity area affects the decision maker's choice, but she makes the same choice for all such reports.

Although in the two-sided case I do not analyze formally what happens if it is possible that the decision maker is naive, it is easy to conjecture the properties of the equilibria. Clearly, the probability that *Right* is chosen, $p(\lambda, \rho)$, must satisfy the same properties as $\eta(\lambda, \rho)$ in the above analysis. That is, this probability must increase in ρ in the P_r ambiguity area, decrease in λ in the P_l ambiguity area, and be constant in the double ambiguity area. The presence of naive types, however, would make the behavior of a rational decision maker interesting in the double ambiguity area. The higher proportion of arguments in favor of *Left* she received, the more likely she would be to choose *Right.* Loosely speaking, this means that the decision maker chooses against the advice of the expert if the expert provides very few arguments.

5 Comparative statics

In this section, I analyze how the equilibrium is affected by the parameters of the model, such as the probabilities of different types of the expert and the prior distribution of arguments.

5.1 Varying the probability of facing the persuader

This section analyzes how changes in the probability of facing the persuader affect the agents' utilities and, more generally, the whole equilibrium. First, I look at what happens when the fraction of honest experts becomes negligible and what happens when the expert is honest with probability of almost 1. Second, I analyze how the probability of facing a particular type of the persuader impacts the bias of his reports and the probability of persuading the decision maker.

Since there are three types of expert, it is necessary to specify how the remaining probabilities change when the probability of facing the expert of type i changes. In Proposition 7, I vary the probability of P_r and keep constant the conditional probability of facing the honest type, given that the expert is not P_r. In such a case the shape of the ambiguity area for P_l remains the same.

Proposition 7 *In every continuous equilibrium,* $\lim_{\pi_H \to 1} \lambda_R = 0$, $\lim_{\pi_H \to 1} \rho_L = 0$, *and* $\lim_{\pi_H \to 0} \lambda_R = \bar{\lambda}_R$, $\lim_{\pi_H \to 0} \rho_L = \bar{\rho}_L$, *where* $\bar{\lambda}_R$ *and* $\bar{\rho}_L$ *are such that* $\frac{R}{R+\bar{\lambda}_R} = E[q_R|R]$ *and* $\frac{\bar{\rho}_L}{\bar{\rho}_L+L} = E[q_R|L]$. *With* $\frac{\pi_H}{1-\pi_r}$ *kept constant, as the probability of facing* P_r *decreases,* *i. the reports of P_r become more extreme;*

ii. the utility of P_r increases;

iii. the utility of P_l decreases;

iv. the expected utility of the decision maker increases.

Proposition 7 says that as the probability of facing the honest expert increases, the ambiguity areas for both persuaders disappear, and the equilibrium converges to a fully revealing equilibrium. On the other hand, as the probability of facing the honest expert goes down, λ_R converges to $\bar{\lambda}_R < \infty$ and ρ_L converges to $\bar{\rho}_L < \infty$. This means that the ambiguity areas are always strict subsets of the report space and the equilibrium never becomes a pure babbling equilibrium. The assumption that arguments are verifiable prevents equilibria from becoming completely uninformative.

It is worth noting that even when the expert is a persuader with probability 1, $\pi_H = 0$, revealing unfavorable information can be a part of an equilibrium. In the one-sided case when $\pi_H = 0$, it is possible to construct an equilibrium in which the persuader suppresses the unfavorable information completely, but the decision maker is still skeptical about all reports with a small number of unfavorable arguments. In particular, since she may attach the same belief to all such reports, it does not hurt the persuader to reveal some unfavorable arguments. In a two-sided case the situation is even more interesting. Even if $\pi_H = 0$, there cannot be an equilibrium with full suppression of unfavorable information. Each persuader would like his type to be misperceived by the decision maker so that her skepticism works in his favor; therefore, when the persuaders receive very few favorable arguments, they must pool. Pooling can happen only if none of the persuaders completely suppress unfavorable information.

This finding suggests that in a disclosure game, complete suppression of unfavorable information may not be robust to uncertainty about the preferences of the expert. Shin (1994a) and Shin (1994b) assume that the preferences of the expert are common knowledge and the state space is discrete, and analyze the equilibrium in which only the favorable information is revealed. My paper suggests that when the state space is discrete, this equilibrium survives uncertainty about the expert's type only under very specific distributional assumptions. This will not, however, be the case in general.

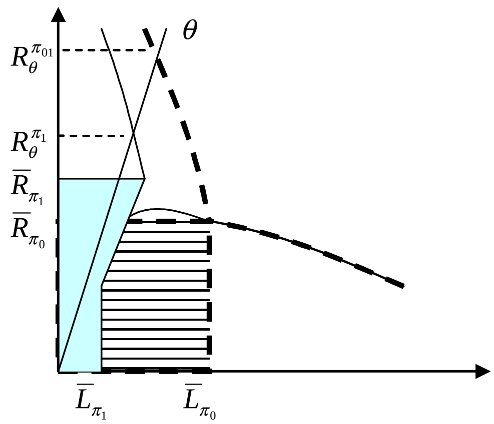


Figure 6: The effect of decreasing π_r, while keeping $\frac{\pi_H}{1-\pi_r}$ constant.

Figure 6 shows how the equilibrium changes as π_r decreases. π_1 and $\pi_0 > \pi_1$ are two different probabilities of facing P_r. The thick curves represent the initial equilibrium in which $\pi_r = \pi_0 = \pi_l$. Since the conditional probability of facing H is kept constant, the shape of the ambiguity area for P_l remains unchanged as π_r changes. As $\pi_r \to 0$, the ambiguity area for P_r becomes smaller, as represented by the thinner curve. The shaded region shows the double ambiguity area for $\pi_r = \pi_1$.

When π_r decreases, the reports of P_r become more extreme. The decision maker becomes less skeptical about the biased reports since they are less likely to come from the persuader; hence, she attaches a higher belief to them. P_r will not send more arguments in favor of *Left,* because this would prove that the quality of *Right* is lower.

From Figure 6 one can see that when the decision maker faces P_r, she chooses *Right* more often when π_r is low (for π_0, a decision maker with θ chooses *Right* whenever $R \geq R_\theta^{\pi_0}$, and for π_1, whenever $R \geq R_\theta^{\pi_1}$). Therefore, the utility of the persuader toward *Right* increases. If the expert happens to be P_l, the decision maker chooses *Right* more often, which decreases the utility of the persuader toward *Left.* However, the utility of the decision maker increases because she is more likely to face the honest expert.

Proposition 7 implies that a financial adviser biased toward a stock that is unpopular among other advisers is better at persuading investors to buy that stock, while a financial adviser biased toward a popular stock is unlikely to successfully promote it.

5.2 Varying the familiarity of the problem

In this section, I analyze how the prior distribution of arguments affects the utility of the decision maker. I focus on the case in which the potential bias of the persuader is known, i.e., when the expert can be either P_r or H.

With the distribution of quality q_R held constant, the distribution of the total number of arguments, $N \equiv L + R$, reflects the decision maker's uncertainty about the choice problem. It describes how the total number of arguments varies from situation to situation for the same decision problem. For example, in each election campaign a different number of issues is important, which can be represented in the model by a relatively dispersed prior belief over N. Other choice problems are likely to be characterized by roughly the same number of arguments every time the decision maker faces them, such us choosing an investment option or buying a car; this is captured by a distribution of N concentrated around the mean. Alternatively, the prior distribution of N may describe the decision maker's knowledge about the problem. An investor with a dispersed distribution of N knows little about the nature of the problem, while an experienced or educated investor is likely to have a concentrated distribution of N.

To isolate the effect of changing the distribution of N while keeping the distribution of the quality of the alternatives unchanged, I reformulate the problem in terms of $(q_R, N) = \left(\frac{R}{R+L}, L + R\right)$, and assume that q_R is uniformly distributed and independent of N. This implies that the joint density of q_R and N is equal to the density of N. Let $G(N; z)$ be the family of distributions with the following properties:

1. the expected value of N is independent of z, $\bar{N} \equiv \int_0^\infty N dG(N; z)$;
2. for all z and for all N we have $G_z(N; z) < 0$ if $N < \bar{N}$ and $G_z(N; z) > 0$ if $N > \bar{N}$;
3. as $z \to \infty$, $G(\cdot)$ becomes degenerate at $\bar{N}$.

Assume that N is distributed according to $G(N; z)$. The second property says that the higher z, the more centered around the mean the distribution is.

Proposition 8 *For every preference type of the decision maker θ and every $\pi > 0$, if $z_1 > z_2$, then the decision maker's utility is higher for $G(N; z_1)$ than for $G(N; z_2)$. As $z \to \infty$, there is full revelation of information.*

Proposition 8 says the lower the uncertainty about N, the better-off the decision maker. When the decision maker knows more about how many arguments are available to the expert, she can more easily infer his information: when she receives a report, she can estimate rather precisely how many arguments have been concealed from her. Given that the dispersion of N can represent the decision maker's familiarity with the choice problem or general uncertainty about it, this proposition implies that the decision maker is better-off in familiar situations with constant complexity.

6 Conclusion

This paper proposes a model of communication that captures many economically relevant situations. In the model messages available to the expert are verifiable, but the expert cannot prove whether he has disclosed all information. As a result, unravelling fails.

The decision maker takes balanced reports at face value, but is skeptical about the unbalanced ones. In such a case, she chooses the alternative favored by the persuader only if the expert can provide her with sufficiently many favorable arguments. If the decision maker does not know the bias of the expert, she ignores his recommendation unless he can reveal sufficiently many arguments. She also does better if she knows more about the complexity of the problem, and is more likely to be persuaded in complex situations.

This paper contributes to the research on whether mandatory disclosure improves the decision maker's welfare by demonstrating that the answer is positive. Disclosure is always beneficial and has the greatest value when the decision maker is unfamiliar with the problem and the complexity of the problem is volatile. However, mandatory disclosure may be difficult to implement if it is difficult to prove that the expert was informed in the first place. This model suggests that a policy of educating the decision maker about which arguments, facts, or characteristics are relevant will improve the decision maker's welfare.

To prove the relevance of the results of this paper in accordance with my model, I should also mention some limitations of it. The model assumes that the decision maker is uninformed, but it would clearly be interesting to analyze the case in which the decision maker has some prior information. Experiments on mass communication indicate that two-sided arguments are more effective when the audience is initially opposed to the expert's position, while one-sided arguments are more successful with listeners who are already disposed toward the expert's position.[12] Moreover, if the audience is later provided with arguments favoring the other position, those who were previously exposed to two-sided argumentation are less likely to be swayed away from this position than those initially exposed to one-sided argumentation.[13] These issues could be addressed within my model if we allow the experts to be imperfectly informed and endow the decision maker with some arguments.

The model presented in this paper provides a good starting point for analyzing communication in more elaborate settings. Because of the argument structure, the differences in information that players have is easily defined, which makes this model especially well-suited for analyzing two-sided communication. Moreover, since disclosing arguments requires time, the model has some natural timing structure built in, which means that it can be applied to dynamic communication.

A Appendix

Proof of Proposition 1

Assume that babbling equilibrium exists. Then all reports must generate the same belief, equal to the prior: $\eta(\lambda, \rho) = E\left[\frac{R}{R+L}\right] \equiv \bar{\eta}$ for all (λ, ρ). Pick a small λ_0 and consider a set $X(\rho_0) = \{(\lambda, \rho) : \lambda \in (0, \lambda_0) \text{ and } \rho \in (\rho_0, \infty)\}$. All reports in this set have very high proportion of arguments in favor of *Right,* and as $\rho_0 \to \infty$, $E\left[\frac{R}{R+L} | (L, R) \in X(\rho_0)\right] \to 1$. Hence, it cannot be the case that this reports always reflect the true state of nature. To generate $\eta(\lambda, \rho) \equiv \bar{\eta}$ for all $(\lambda, \rho) \in X(\rho_0)$ there must exist a set of states $Y(\rho_0)$ in which quality of *Right* is low, such that the persuader P_r sends reports from $X(\rho_0)$ when $(L, R) \in Y(\rho_0)$. Below, I show that for a sufficiently high ρ_0, it is impossible to find a set of states $Y(\rho_0)$ in which reports from $X(\rho_0)$ are feasible such that if P_r sends reports from $X(\rho_0)$ whenever $(L, R) \in Y(\rho_0)$ the belief that the decision maker forms is at least $\bar{\eta}$. This is simply

[12] See Hovland, Lumsdaine, and Sheffield (1949).
[13] See Lumsdaine and Janis (1953).

because as ρ_0 increases, states with low quality of *Right* in which $X(\rho_0)$ is feasible become relatively less likely than states in $X(\rho_0)$, which implies that upon seeing a report from $X(\rho_0)$ the decision maker beliefs that it came from the honest expert with probability close to 1.

Take a set $Y(\rho_0) = \left\{(L,R) : R \geq \rho_0, L \geq \frac{1-\bar{\eta}}{\bar{\eta}}R\right\}$. This set contains all states, and only these states, in which the quality of *Right* is lower than $\bar{\eta}$. If we are unable to generate a belief equal to or below $\bar{\eta}$ for such $Y(\rho_0)$, then we are unable to do this for any other set.

The expected quality of *Right* given that the report is in X is

$$
\begin{aligned}
E\left[\frac{R}{R+L}|(\lambda,\rho)\in X\right] &= \Pr(H|(\lambda,\rho)\in X)\,E\left[\frac{R}{R+L}|(L,R)\in X\right] \\
&\quad +\Pr(P_r|(\lambda,\rho)\in X)\,E\left[\frac{R}{R+L}|(L,R)\in Y\right].
\end{aligned}
$$

Let $f^R(\cdot)$ denote the marginal p.d.f. of R. We have

$$
\begin{aligned}
\lim_{\rho_0\to\infty}\frac{\int_{\rho_0}^{\infty}\int_{\frac{1-\bar{\eta}}{\bar{\eta}}R}^{\infty}\frac{R}{R+L}f^L(L|R)\,f^R(R)\,dLdR}{\int_{\rho_0}^{\infty}\int_0^{\lambda_0}f^L(L|R)\,f^R(R)\,dLdR} &\leq \lim_{\rho_0\to\infty}\frac{\int_{\rho_0}^{\infty}\int_{\frac{1-\bar{\eta}}{\bar{\eta}}R}^{\infty}f^L(L|R)\,f^R(R)\,dLdR}{\int_{\rho_0}^{\infty}\int_0^{\lambda_0}f^L(L|R)\,f^R(R)\,dLdR} =^H= \\
&= \lim_{\rho_0\to\infty}\frac{\int_{\frac{1-\bar{\eta}}{\bar{\eta}}\rho_0}^{\infty}f(L|\rho_0)\,dLf^R(\rho_0)}{\int_0^{\lambda_0}f^L(L|\rho_0)\,dLf^R(\rho_0)} \\
&= \lim_{\rho_0\to\infty}\frac{1-F^L\left(\frac{1-\bar{\eta}}{\bar{\eta}}\rho_0|\rho_0\right)}{F^L(\lambda_0|\rho_0)} = 0,
\end{aligned}
$$

where $=^H=$ denotes that l'Hospital rule was applied and the last equality comes from the regularity conditions 2. Since $\frac{dF^L(L|R)}{dR} \geq 0$, we have $\lim_{\rho_0\to\infty}F^L(\lambda_0|\rho_0) > 0$, while $\lim_{\rho_0\to\infty}F^L\left(\frac{1-\bar{\eta}}{\bar{\eta}}\rho_0|\rho_0\right) = 1$.

Therefore, we have

$$
\begin{aligned}
&\lim_{\rho_0\to\infty}\Pr(H|(\lambda,\rho)\in X) \\
= &\lim_{\rho_0\to\infty}\frac{(1-\pi)\int_{\rho_0}^{\infty}\int_0^{\lambda_0}f^L(L|R)\,f^R(R)\,dLdR}{(1-\pi)\int_{\rho_0}^{\infty}\int_0^{\lambda_0}f^L(L|R)\,f^R(R)\,dLdR+\pi\int_{\rho_0}^{\infty}\int_{\frac{1-\bar{\eta}}{\bar{\eta}}R}^{\infty}f^L(L|R)\,f^R(R)\,dLdR} = 1,
\end{aligned}
$$

$$\lim_{\rho_0\to\infty} \Pr\left(P_r|\left(\lambda,\rho\right)\in X\right) E\left[\frac{R}{R+L}|\left(L,R\right)\in Y\right]$$

$$= \lim_{\rho_0\to\infty} \frac{\pi\int_{\rho_0}^{\infty}\int_{\frac{1-\bar{\eta}}{\bar{\eta}}R}^{\infty}\frac{R}{R+L}f^L\left(L|R\right)f^R\left(R\right)dLdR}{\left(1-\pi\right)\int_{\rho_0}^{\infty}\int_0^{\lambda_0}f^L\left(L|R\right)f^R\left(R\right)dLdR+\pi\int_{\rho_0}^{\infty}\int_{\frac{1-\bar{\eta}}{\bar{\eta}}R}^{\infty}f^L\left(L|R\right)f^R\left(R\right)dLdR} = 0,$$

hence

$$\lim_{\rho_0\to\infty} E\left[\frac{R}{R+L}|\left(\lambda,\rho\right)\in X\right] = 1 > \bar{\eta},$$

which completes the proof. ■

Proof of Proposition 2

Let $\sigma_i\left(L,R\right)$ denote the set of all reports that lie in the support of the strategy of an expert of type i, that is, $\sigma_i\left(L,R\right) = \left\{\left(\lambda,\rho\right) : m_i\left(\left(\lambda,\rho\right)|\left(L,R\right)\right) > 0\right\}$. Recall that $\eta\left(\lambda,\rho\right) = E\left[\frac{R}{R+L}|\left(\lambda,\rho\right)\right]$ and $Z\left(\lambda,\rho\right) = \left(\left(L,R\right) : L \geq \lambda \text{ and } R \geq \rho\right)$ is a set of all states in which a report $\left(\lambda,\rho\right)$ is feasible.

Step 1 The presence of the honest expert, H, assures that there are no off-equilibrium beliefs: whenever $\left(\lambda,\rho\right) \notin \sigma_{P_r}\left(L,R\right)$ for all $\left(L,R\right)$ (that is, whenever $\left(\lambda,\rho\right)$ is never sent by P_r) then $\eta\left(\lambda,\rho\right) = \frac{\rho}{\lambda+\rho}$.

Step 2 $\eta\left(0,\rho\right) < 1$ for all ρ.

By step 1, $\eta\left(\lambda,\rho\right) < 1$ for all $\lambda > 0$. Assume there exist ρ such that $\eta\left(0,\rho\right) = 1$. Then for all $\left(L,R\right) \in Z\left(0,\rho\right)$, P_r is able to generate a belief equal to 1, which would violate Bayes' rule.

Step 3 For all ρ, there exists $l_\rho > 0$ such that $\eta\left(\lambda,\rho\right) = \eta\left(0,\rho\right)$ for all $\lambda < l_\rho$.

Assume not. This means that for some ρ and for all $\varepsilon > 0$, we can find $\lambda_0 < \varepsilon$ such that $\eta\left(\lambda_0,\rho\right) \neq \eta\left(0,\rho\right)$. Assume first that $\eta\left(\lambda_0,\rho\right) < \eta\left(0,\rho\right)$. By Step 2 $\eta\left(\lambda_0,\rho\right) < \eta\left(0,\rho\right) < 1$. But then P_r always prefers to send $\left(0,\rho\right)$ instead of $\left(\lambda_0,\rho\right)$, which implies that $\left(\lambda_0,\rho\right)$ is sent only by H. Step 1 implies that $\eta\left(\rho,\lambda_0\right) = \frac{\rho}{\lambda_0+\rho}$. But $\eta\left(\rho,\lambda_0\right) = \frac{\rho}{\lambda_0+\rho}$ becomes arbitrarily close to 1 as $\varepsilon \to 0$. This contradicts the assumption that $\eta\left(\lambda_0,\rho\right) < \eta\left(0,\rho\right) < 1$.

Assume then that for all $\varepsilon > 0$ we can find $\lambda_0 < \varepsilon$ such that $\eta\left(\lambda_0,\rho\right) > \eta\left(0,\rho\right)$. Then P_r may send $\left(0,\rho\right)$ only if $L = 0$ because for any $L > 0$ he would find $\lambda_0 \leq L$ and send $\left(\lambda_0,\rho\right)$ instead. But then $\eta\left(0,\rho\right) = 1$, which contradicts Step 2.

Define $\eta^*_\rho \equiv \eta\left(0,\rho\right)$.

Step 4 η^*_ρ is weakly increasing in ρ.

Assume that there exist $\rho_2 > \rho_1$ such that $\eta^*_{\rho_2} < \eta^*_{\rho_1}$. Then P_r never sends $\left(0,\rho_2\right)$, which by Step 1 implies that $\eta^*_{\rho_2} = 1$, which contradicts Step 2.

Step 1 and Step 3 imply that in equilibrium P_r does not always completely suppress unfavorable information; otherwise $\eta(\lambda, \rho)$ would be a strictly decreasing function of λ.

Step 5 η^*_ρ cannot be constant

The argument is similar to the proof of Proposition **2.** The argument would not hold if $\lambda_\rho \to 0$ as $\rho \to \infty$. But this cannot be the case, because then either $\eta(\lambda_\rho + \varepsilon, \rho) > \eta^*_\rho$ or $\eta(\lambda_\rho + \varepsilon, \rho) < \eta^*_\rho$ for an arbitrarily small $\varepsilon > 0$. The former cannot be the case because then the persuader would never send $(0, \lambda)$. The latter cannot be the case because then the persuader would not send $(\lambda_\rho + \varepsilon, \rho)$, and therefore the decision maker would take this report at face value, that is $\eta(\lambda_\rho + \varepsilon, \rho) = \frac{\rho}{\lambda_\rho + \varepsilon + \rho}$. But $\eta(\lambda_\rho + \varepsilon, \rho) = \frac{\rho}{\lambda_\rho + \varepsilon + \rho} \to 1 > \eta^*_\rho$ as $\lambda_\rho \to 0$, which is a contradiction.

Step 6 $\eta^*_\rho \geq \frac{\rho}{\rho + \lambda_\rho}$

Assume $\frac{\rho}{\rho + \lambda_\rho} > \eta^*_\rho$. Then for all $\varepsilon > 0$, we can find $\lambda_0 \in (\lambda_\rho, \lambda_\rho + \varepsilon)$ such that $\eta(\lambda_0, \rho) \neq \eta^*_\rho$. Assume first that $\eta(\lambda_0, \rho) < \eta^*_\rho$ Then only H sends (λ_0, ρ), and therefore, $\eta(\lambda_0, \rho) = \frac{\rho}{\rho + \lambda_0} > \eta^*_\rho$, a contradiction. Assume now that $\eta(\lambda_0, \rho) > \eta^*_\rho$. But then (λ_0, ρ) is more attractive to P_r than any report of a form $(\lambda \leq \lambda_\rho, \rho)$. Therefore, P_r will send (λ_ρ, ρ) only if (L, R) is such that $\frac{R}{R+L} \geq \frac{\rho}{\rho + \lambda_\rho} > \eta^*_\rho$, which contradicts $\eta(\lambda_\rho, \rho) = \eta^*_\rho$.

Step 7 Let $\hat{\rho} = \max\left\{\rho : \eta^*_\rho = \hat{\eta}\right\}$. Then $\lambda_{\hat{\rho}}$ is such that $\frac{\hat{\rho}}{\hat{\rho} + \lambda_{\hat{\rho}}} = \hat{\eta}$.

By Step 6 we have $\frac{\hat{\rho}}{\hat{\rho} + \lambda_{\hat{\rho}}} \leq \hat{\eta}$. Assume then $\frac{\hat{\rho}}{\hat{\rho} + \lambda_{\hat{\rho}}} < \hat{\eta}$. To generate a belief $\hat{\eta}$ for a report $(\lambda_{\hat{\rho}}, \hat{\rho})$ it must be that there exists $(L_1, R_1) \in Z(\lambda_{\hat{\rho}}, \hat{\rho})$ such that $\frac{R_1}{R_1 + L_1} \geq \hat{\eta}$ and $\sigma_{P_r}(L_1, R_1) = (\lambda_{\hat{\rho}}, \hat{\rho})$. But that implies that $R_1 > \hat{\rho}$, and by definition of $\hat{\rho}$ we know $\eta^*_{R_1} > \hat{\eta}$, therefore, P_r would prefer to send $(0, R_1)$ to sending $(\lambda_{\hat{\rho}}, \hat{\rho})$. A contradiction. ■

Proof of Proposition 3

I characterize all equilibria in which the belief function $\eta(\lambda, \rho)$ is continuous in λ and ρ. First, continuity requires that $\eta^*_\rho = \frac{\rho}{\lambda_\rho + \rho}$; therefore, there exists a unique λ_ρ for a given η^*_ρ. This, together with continuity, implies that there is no $\lambda > \lambda_\rho$ such that $\eta(\lambda, \rho) \geq \eta^*_\rho$, which implies that only H sends reports of a form $(\lambda > \lambda_\rho, \rho)$. Steps 1-4 from the proof of Proposition 2 together with $\eta^*_\rho = \frac{\rho}{\lambda_\rho + \rho}$ imply that all equilibria have a shape like the one in Figure 4. The solid curve represents λ_ρ for each ρ. Given that $\eta^*_\rho = \frac{\rho}{\lambda_\rho + \rho}$ the equilibrium depicted above is characterized by a strictly increasing η^*_ρ but for $\rho \in (\rho_1, \rho_2)$. Now, I show that $\eta^*_{\rho_1} < \eta^*_{\rho_2}$, hence this equilibrium is not continuous. Therefore in a continuous equilibrium η_ρ^* must be strictly increasing.

First, we can find ε such that η_ρ^* is strictly increasing in $\rho^{'}$ for all $\rho' \in (\rho_1, \rho_1 + \varepsilon)$ and $\rho' \in (\rho_2 - \varepsilon, \rho_2)$. If we could not, the equilibrium would not be continuous. For $R = \rho'$ the persuader reveals R and sends some $\lambda < \lambda_{\rho'}$; therefore, the belief function for these ρ's is defined by

$$\begin{aligned} \eta_{\rho'}^* &= \Pr(H|\rho\prime, \lambda \leq \lambda_{\rho'}) E[q_R|R = \rho\prime, L \leq \lambda_{\rho'}] \\ &\quad + \Pr\left(P_r|\rho\prime, \lambda \leq \lambda_{\rho\prime}\right) E[q_R|R = \rho\prime]. \end{aligned}$$

where $\lambda_{\rho'} = \frac{1-\eta_{\rho'}}{\eta_{\rho'}}\rho'$. If $\eta(\rho, \lambda)$ is continuous then $\lim_{\varepsilon\to 0} \eta_{\rho_2+\varepsilon}^* = \lim_{\varepsilon\to 0} \eta_{\rho_1-\varepsilon}^*$.

Let $g(\rho)$ be η_ρ^* that satisfied the above equation for each ρ. This equation can be rewritten as:

$$\begin{aligned} Y \equiv g(\rho)\left(F^L\left(\frac{1-g(\rho)}{g(\rho)}\rho|R = \rho\right)(1-\pi) + \pi\right) + \\ -(1-\pi)\int_0^{\frac{1-g(\rho)}{g(\rho)}\rho} \frac{\rho f^L(L|R = \rho)}{\rho + L} dL \quad -\pi \textstyle\int_0^\infty \frac{\rho}{\rho+L} f^L(L|R = \rho)\, dL = 0. \end{aligned} \tag{8}$$

Clearly, $g(\rho_2) = \lim_{\varepsilon\to 0} \eta_{\rho_2+\varepsilon}^* = \eta_{\rho_2}^*$ and $g(\rho_1) = \lim_{\varepsilon\to 0} \eta_{\rho_1-\varepsilon}^* = \eta_{\rho_1}^*$. I now show that $\frac{dg(\rho)}{d\rho} > 0$, which implies that $g(\rho_2) > g(\rho_1)$, which in turn means that $\eta_{\rho_2}^* > \eta_{\rho_1}^*$, which is a contradiction.

By the implicit function theorem $\frac{dg(\rho)}{d\rho} = -\frac{\frac{\partial Y}{\partial \rho}}{\frac{\partial Y}{\partial g}}$, and

$$\frac{\partial Y}{\partial g} = F^L\left(\frac{1-g}{g}\rho|R = \rho\right)(1-\pi) + \pi > 0,$$

$$\begin{aligned} \frac{\partial Y}{\partial \rho} &= -(1-\pi)\int_0^{\frac{1-g}{g}\rho} \frac{L f^L(L|R = \rho)}{(\rho + L)^2} dL - \pi \int_0^\infty \frac{L f^L(L|R = \rho)}{(\rho + L)^2} dL \\ &\quad -(1-\pi)\int_0^{\frac{1-g}{g}\rho} \frac{\rho F_R^L(L|R = \rho)}{(\rho + L)^2} dL - \pi \int_0^\infty \frac{\rho F_R^L(L|R = \rho)}{(\rho + L)^2} dL, \end{aligned}$$

where the expression above was obtained by applying first integration by parts to the formula for Y, taking the derivative and applying integration by parts again. Using the regularity condition (2) we get $\frac{dY}{d\rho} < 0$, which in turn implies that $\frac{dg(\rho)}{d\rho} > 0$.

When η_ρ^* is strictly increasing, P_r sends always $\rho = R$. Given this, I sometimes use η_R^* instead of η_ρ^*.

It remains to show that for each R, η_R^* described by equation (3) exists and is unique. Equation (3) can be rewritten as:

$$\eta_R^* = \frac{\pi \int_0^\infty \frac{R}{R+L} f^L(L|R)\, dL + (1-\pi) \int_0^{\frac{1-\eta_R^*}{\eta_R^*}R} \frac{R}{R+L} f^L(L|R)\, dL}{\pi + (1-\pi) F^L\left(\frac{1-\eta_R^*}{\eta_R^*}R|R\right)}. \tag{9}$$

The left hand side (LHS) goes from 0 to 1. The right hand side (RHS) is continuous, and we have $RHS(\eta_R^* \to 0) \to \bar{\eta}_R > 0$ and $RHS(\eta_R^* = 1) = \bar{\eta}_R < 1$, where $\bar{\eta}_R = \int_0^\infty \frac{R}{R+L} f^L(L|R)\, dL$; therefore, the solution to equation (9) exists.

The LHS is strictly increasing. If we differentiate the RHS with respect to η_R^*, we obtain

$$\begin{aligned} \frac{dRHS}{d\eta_R^*} &= \frac{(1-\pi) f^L\left(\frac{1-\eta_R^*}{\eta_R^*}R|R\right)}{\left(\pi + (1-\pi) F^L\left(\frac{1-\eta_R^*}{\eta_R^*}R|R\right)\right)^2} \frac{-1}{(\eta_R^*)^2} R \cdot \\ &\cdot \left(\pi(\eta_R^* - \bar{\eta}_R) + (1-\pi)\left(\eta_R^* F^L\left(\frac{1-\eta_R^*}{\eta_R^*}R|R\right) - \int_0^{\frac{1-\eta_R^*}{\eta_R^*}R} \frac{Rf^L(L|R)}{R+L} dL\right)\right). \end{aligned}$$

Evaluating it at η_R^* that satisfies equation (9) we obtain

$$\frac{dRHS}{d\eta_R^*} = \frac{(1-\pi) f^L\left(\frac{1-\eta_R^*}{\eta_R^*}R|R\right)}{\left(\pi + (1-\pi) F^L\left(\frac{1-\eta_R^*}{\eta_R^*}R|R\right)\right)} (\eta^* - \eta^*) = 0.$$

This implies that the solution to equation (9) is unique. ■

Proof of Proposition 4

This proof follows exactly the proof of Proposition 2 with $p(\lambda, \rho)$ replacing $\eta(\lambda, \rho)$. The existence of H assures that there are no off-equilibrium beliefs: whenever $(\lambda, \rho) \notin \sigma_{P_r}(L, R)$ for some (L, R) then $\eta(\lambda, \rho) = \frac{\rho}{\lambda+\rho}$; and therefore $p(\lambda, \rho) = \frac{\rho}{\lambda+\rho}$. First, we by the same logic as in proof of Proposition 2 we have $p(\lambda, \rho) < 1$ for all reports.

Now we are ready to show that for all ρ, there exists $\lambda_\rho > 0$ such that $p(\lambda, \rho) = p(0, \rho)$ for all $\lambda < \lambda_\rho$. Assume not. Then for each ρ and for all $\varepsilon > 0$, we can find $\lambda_0 < \varepsilon$ such that $p(\lambda_0, \rho) \neq p(0, \rho)$. Assume first that $p(\lambda_0, \rho) < p(0, \rho) < 1$. Then

P_r always prefers to send $(0, \rho)$ instead of (λ_0, ρ), which implies that (λ_0, ρ) is sent only by H. Then $p(\rho, \lambda_0) = \frac{\rho}{\lambda_0 + \rho}$. But $p(\rho, \lambda_0) = \frac{\rho}{\lambda_0 + \rho} > \frac{\rho}{\varepsilon + \rho}$ becomes arbitrarily close to 1 as $\varepsilon \to 0$. This contradicts the assumption that $p(\lambda_0, \rho) < p(0, \rho) < 1$.

Assume now that for all $\varepsilon > 0$ we can find $\lambda_0 < \varepsilon$ such that $p(\lambda_0, \rho) > p(0, \rho)$. Then P_r may send $(0, \rho)$ only if $L = 0$ because for any $L > 0$ he would find such $\lambda_0 \leq L$ and send (λ_0, ρ) instead. But then $p(0, \rho) = 1$, which is a contradiction.

The fact that $p(\lambda, \rho)$ is constant for small λ implies that the rational decision maker's belief must be continuous in λ for $\lambda \leq \lambda_\rho$. Differentiating $p(\lambda, \rho)$ (equation 5) with respect to λ we get

$$g(\eta(\lambda, \rho)) \frac{d\eta(\lambda, \rho)}{d\lambda} = \frac{\mu}{1-\mu} g\left(\frac{\rho}{\rho + \lambda}\right) \frac{\rho}{(\rho + \lambda)^2},$$

which implies that the belief of the rational decision maker is strictly increasing in λ for $\lambda \leq \lambda_\rho$. ■

Proof of Proposition 5

This proof shows that for any fixed R, all types of the decision maker are at least as well-off in expectation in the continuous equilibria as in any discontinuous one. Let $\eta_R^{*c} \equiv \eta(0, R)$ in any continuous equilibrium and let $\eta_R^* \equiv \eta(0, R)$ in some discontinuous equilibrium. Recall that λ_R is the highest λ such that $\eta(\lambda, R) = \eta_R^*$ for all $\lambda < \lambda_R$, and let λ_R^c be the highest λ such that $\eta(\lambda, R) = \eta_R^{*c}$ for all $\lambda < \lambda_R^*$. From the proof of Propositions 2 and 3 we know that $\frac{R}{R + \lambda_R^c} = \eta_R^{*c}$ and $\frac{R}{R + \lambda_R} \leq \eta_R^*$. Clearly, for a given R and keeping η_R^* fixed, the decision maker is weakly better-off when $\frac{R}{R + \lambda_R} = \eta_R^*$ than when $\frac{R}{R + \lambda_R} < \eta_R^*$, as in both cases she makes the same decision when facing P_r, but in the latter case she may make worse decisions when facing H. Hence I assume that $\frac{R}{R + \lambda_R} = \eta_R^*$, and show that even in this best scenario for the discontinuous equilibria the decision maker still prefers the continuous ones.

First, recall that η_R^{*c} is defined as x that solves the following equation:

$$x = \frac{\pi E[q_R | R] + (1 - \pi) \Pr(q_R > x) E[q_R | q_R > x, R]}{\pi + (1 - \pi) \Pr(q_R > x | R)}.$$

At the end of the proof of Proposition 3 I have shown that $RHS < LHS$ for $x < \eta_R^{*c}$

and $RHS > LHS$ for $x > \eta_R^{*c}$; in other words

$$\frac{\pi E\left[q_R|R\right] + (1-\pi)\Pr\left(q_R > x\right) E\left[q_R|q_R > x, R\right]}{\pi + (1-\pi)\Pr\left(q_R > x|R\right)} \begin{cases} > x \text{ if } x < \eta_R^{*c} \\ < x \text{ if } x > \eta_R^{*c} \end{cases}. \tag{10}$$

For each R, we have three cases: 1. $\eta_R^* < \eta_R^{*c}$, 2. $\eta_R^* > \eta_R^{*c}$ or 3. $\eta_R^* = \eta_R^{*c}$.

Case 1: $\eta_R^* < \eta_R^{*c}$. A decision maker with $\theta \geq \eta_R^{*c}$ or with $\theta \leq \eta_R^*$ makes the same decision in both equilibria. For a given R a decision maker with $\theta \in (\eta_R^*, \eta_R^{*c})$ chooses *Left* in the discontinuous equilibrium for all L, but *Right* in the continuous equilibria when the expert is a persuader for all L, or when the expert is honest and $L > \theta$. The expected quality q_R conditional on the events in which she chooses *Right* in the continuous equilibrium is

$$\frac{\pi E\left[q_R|R\right] + (1-\pi) P\left(q_R > \theta\right) E\left[q_R|q_R > \theta, R\right]}{\pi + (1-\pi) P\left(q_R > \theta|R\right)},$$

which by equation 10 is greater than θ, hence *Right* is the better choice. This means that the decision maker is better off in the continuous equilibrium for this R.

Case 2: $\eta_R^* > \eta_R^{*c}$. A decision maker with $\theta \leq \eta_R^{c*}$ or with $\theta \geq \eta_R^*$ makes the same decision in both equilibria. For a given R a decision maker with $\theta \in \left(\eta_\rho^{*c}, \eta_\rho^*\right)$ chooses *Right* in the discontinuous equilibrium for all L, but *Left* in the continuous equilibria when the expert is a persuader for all L, or when the expert is honest and $L > \theta$. The expected quality q_R conditional on the events in which she chooses *Left* in the continuous equilibrium is

$$\frac{\pi E\left[q_R|R\right] + (1-\pi) P\left(q_R > \theta\right) E\left[q_R|q_R > \theta, R\right]}{\pi + (1-\pi) P\left(q_R > \theta|R\right)},$$

which by equation 10 is smaller than θ, hence *Left* is the better choice. This means the decision maker is again better off in the continuous equilibrium for this R.

Case 3: $\eta_R^* = \eta_R^{*c}$. It is immediate that the decision maker is indifferent between the equilibria. ■

Proof of Proposition 6

Step 1 If for some (λ, ρ) there exists no (L, R) such that $(\lambda, \rho) \in \sigma_{P_r}(L, R)$ or $(\lambda, \rho) \in \sigma_{P_l}(L, R)$ then $\eta(\lambda, \rho) = \frac{\lambda}{\lambda+\rho}$.

This follows directly from the presence of H type. This finding will be used extensively in the proof.

Step 2 $\eta(0,\rho) < 1$ for all ρ and $\eta(\lambda,0) > 0$ for all λ.

The proof of the first part is identical to Step 2 of the proof of Proposition 2. By a similar argument we get $\eta(\lambda,0) > 0$.

Step 3 i) For all ρ, there exists $\lambda_\rho > 0$ such that $\eta(\lambda,\rho) = \eta(0,\rho)$ for all $\lambda < \lambda_\rho$.

ii) For all λ, there exists $\rho_\lambda > 0$ such that $\eta(\lambda,\rho) = \eta(\lambda,0)$ for all $\rho < \rho_\lambda$

Assume that part (i) does not hold, which means that there exists $\lambda_0 > 0$ such that $\eta(\lambda,\rho)$ is strictly increasing or decreasing in λ for $\lambda < \lambda_0$. If $\eta(\lambda,\rho)$ is decreasing in λ then P_r always prefers to send $(0,\rho)$ instead of any (λ,ρ) with $\lambda < \lambda_0$ and P_l prefers to send as high λ as possible for $\lambda \in (0,\lambda_0)$. Take (ε,ρ) with $\varepsilon < \lambda_0$. P_l may send (ε,ρ) only if $L = \varepsilon$ and $R \geq \rho$, therefore it must be that $\eta(\varepsilon,\rho) \geq \frac{\rho}{\rho+\varepsilon}$. But $\eta(\varepsilon,\rho) \geq \frac{\rho}{\rho+\varepsilon} \to_{\varepsilon\to 0} 1 > \eta(0,\rho)$, which contradicts the continuity assumption.

Assume now that (λ,ρ) is strictly increasing in λ for $\lambda \leq \lambda_0$. Then P_l never sends (ε,ρ) for small ε : he would prefer to send $(0,\rho)$ instead. P_r may send (ε,ρ) only if $L = \lambda_\rho$ and $R \geq \rho$. But then $\eta(\varepsilon,\rho) \geq \frac{\rho}{\rho+\varepsilon} \to_{\varepsilon\to 0} 1$, which again contradicts the assumption of continuity of $\eta(\lambda,\rho)$. A similar argument holds for part ii).

Define $\eta^*_\rho \equiv \eta(0,\rho)$ and $\eta^*_\lambda \equiv \eta(\lambda,0)$.

Step 4 i) P_r sends only reports of the form $(\lambda,\rho) : \lambda \leq \lambda_\rho$,

ii) P_l sends only reports of the form $(\lambda,\rho) : \rho \leq \rho_\lambda$.

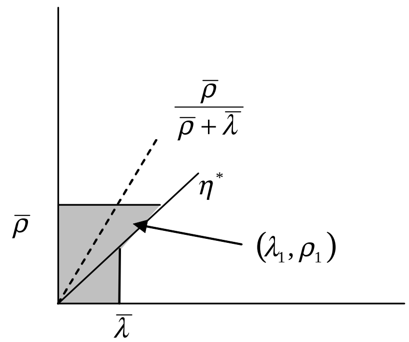
Assume that P_r sends (λ_0,ρ_0), where $\lambda_0 > \lambda_{\rho_0}$. This means that $\eta(\lambda_0,\rho_0) \geq \eta^*_{\rho_0}$. This in turn means that for some $\lambda \in \left(\lambda_{\rho_0},\lambda_0\right)$, the belief function $\eta(\lambda,\rho_0)$ is strictly increasing in λ but $\eta(\lambda,\rho_0) < \eta^*_{\rho_0}$. But these reports would be sent by H only, which means that $\eta(\lambda,\rho_0) = \frac{\rho_0}{\lambda+\rho_0}$. This contradicts that $\eta(\lambda,\rho_0)$ was increasing in λ. An analogous proof holds for part (ii).

By the same arguments we have $\eta^*_\rho > \eta(\lambda,\rho)$ for all $\lambda > \lambda_\rho$ and $\eta^*_\lambda < \eta(\lambda,\rho)$ for all $\rho > \rho_\lambda$.

Step 5 Previous steps imply that there exist $\bar{\rho} > 0$ and $\bar{\lambda} > 0$ such that $\eta^*_\rho = \eta^*_\lambda \equiv \eta^*$ for all $\rho \leq \bar{\rho}$ and for all $\lambda \leq \bar{\lambda}$. If we take the highest such $\bar{\rho}$ and $\bar{\lambda}$, then P_l never sends reports $(0,\rho > \bar{\rho})$ and P_r never sends reports $\left(\lambda > \bar{\lambda},0\right)$. By the same argument as in the proof of Proposition 3 we have that η^*_ρ is strictly increasing in ρ for $\rho > \bar{\rho}$, and η^*_λ is strictly decreasing in λ for $\lambda > \bar{\lambda}$. Moreover, by continuity of $\eta(\lambda,\rho)$ for $\rho > \bar{\rho}$ we have $\eta^*_\rho = \frac{\rho}{\rho+\lambda_\rho}$ and for $\lambda > \bar{\lambda}$ we have $\eta^*_\lambda = \frac{\rho_\lambda}{\rho_\lambda+\lambda}$. The definition of η^*_ρ is the same as in equation (3), η^*_λ is defined analogously and they are unique by the same argument as in Proposition **3** .

Step 6 There are three possible situations, $\eta^* > \frac{\bar{\rho}}{\bar{\rho}+\bar{\lambda}}$, $\eta^* < \frac{\bar{\rho}}{\bar{\rho}+\bar{\lambda}}$ or $\eta^* = \frac{\bar{\rho}}{\bar{\rho}+\bar{\lambda}}$. Below I describe the shape of the equilibrium if $\eta^* < \frac{\bar{\rho}}{\bar{\rho}+\bar{\lambda}}$. For the remaining cases the discussion is analogous.

If $\eta^* < \frac{\bar{\rho}}{\bar{\rho}+\hat{\lambda}}$, then for all (λ, ρ) that lie in the shaded area in the figure below, and only for these reports (or when $\frac{R}{L+R} = \eta^*$), we have $\eta(\lambda, \rho) = \eta^*$.



To see this notice that by Step 3 $\eta(\lambda, \rho) = \eta^*$ for all $(\lambda, \rho) : \lambda \leq \bar{\lambda}$ and $\rho \leq \bar{\rho}$. Take report $(\lambda_1, \rho_1) : \frac{\rho_1}{\rho_1+\lambda_1} > \eta^*$ like in the figure above. By Step 4, $\eta(\lambda_1, \rho_1) \leq \eta^*$. Assume that $\eta(\lambda_1, \rho_1) < \eta^*$; then only P_l may send (λ_1, ρ_1), which by Step 4 implies that $\eta(\lambda_1, \rho_1) = \eta^*_{\lambda_1}$, and by Step 5 η^*_λ is strictly decreasing in λ, which in turn implies that $(\lambda_1, \rho_1) \in \sigma_{P_l}(L, R)$ only if $L = \lambda_1$ and $R \geq \rho_1$. But this implies that $\eta(\lambda_1, \rho_1) \geq \frac{\rho_1}{\rho_1+\lambda_1} > \eta^*$, which is a contradiction.

Step 6 allows us to summarize the shape of any equilibrium. If $\eta^* < \frac{\bar{\rho}}{\bar{\rho}+\bar{\lambda}}$, then the equilibrium looks like in the figure below.

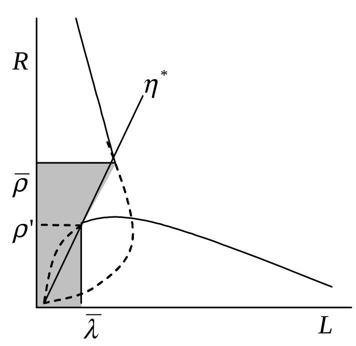


Figure A

The grey area represents all reports that generate the same belief η^*. The solid curves represent the areas in which only P_r and H (along the vertical axis) or only P_l and

H (along the horizontal axis) send reports.

In what follows below I use $\bar{R}$ instead of $\bar{\rho}$ and $\bar{L}$ instead of $\bar{\lambda}$. In the proof of Proposition 3 I have shown that η_ρ^* (and therefore also η_λ^*) exists and is unique and strictly increasing for $\rho > \bar{R}$ (η_λ^* is strictly decreasing for $\lambda > \bar{L}$). I will show now that $\bar{R}$ and $\bar{L}$ and η^* are unique. Let all reports (λ, ρ) that generate η^* and either $\rho \leq \bar{R}$ or $\lambda \leq \bar{L}$ be called the double ambiguity area (DAA).

First, by continuity of $\eta(\lambda, \rho)$, $\bar{R}$ and $\bar{L}$ must be such that $\eta^*_{\bar{R}+\varepsilon} \to \eta^*$ and $\eta^*_{\bar{L}+\varepsilon} \to \eta^*$ as $\varepsilon \to 0$. This means that $\bar{R}$ and $\bar{L}$ must satisfy equations (6) and (7), which can be rewritten as

$$\eta^* = \frac{\frac{\pi_r}{1-\ \pi_l} \int_0^{1-\bar{R}} \frac{\bar{R}}{\bar{R}+L} f^L\left(L|\bar{R}\right) dL + \frac{\pi_H}{1-\ \pi_l} \int_0^{\lambda_{\bar{R}}} \frac{\bar{R}}{\bar{R}+L} f^L\left(L|\bar{R}\right) dL}{\frac{\pi_r}{1-\ \pi_l} + \frac{\pi_H}{1-\ \pi_l} F\left(\lambda_{\bar{R}}|\bar{R}\right)}, \tag{11}$$

$$\eta^* = \frac{\frac{\pi_l}{1-\ \pi_r} \int_0^{1-\bar{L}} \frac{R}{R+\bar{L}} f^R\left(R|\bar{L}\right) dR + \frac{\pi_H}{1-\ \pi_r} \int_0^{\rho_{\bar{L}}} \frac{R}{R+\bar{L}} f^R\left(R|\bar{L}\right) dR}{\frac{\pi_l}{1-\ \pi_r} + \frac{\pi_H}{1-\ \pi_r} F^R\left(\rho_{\bar{L}}|\bar{L}\right)}. \tag{12}$$

These equations uniquely determine $\bar{R}$ and $\bar{L}$ as a function of η^*. For all reports in DAA to generate the same belief η^*, this belief must satisfy

$$\begin{aligned} \eta^* \;=\; & P\left(P_r|DAA\right) E\left[\frac{R}{R+L}|DAA, P_r\right] + \\ & + P\left(P_l|DAA\right) E\left[\frac{R}{R+L}|DAA, P_l\right] + P\left(H|DAA\right) E\left[\frac{R}{R+L}|DAA, H\right]. \end{aligned}$$

Recall Figure A; the equation above can be rewritten as follows (where f is used instead of $f(L, R)$ to shorten the formula):

$$\begin{aligned} \eta^* \;=\; & \frac{\pi_r \int_0^{\bar{R}} \int_0^{1-R} q_R f dL dR + \pi_l \int_0^{\bar{L}} \int_0^{1-L} q_R f dL dR}{\pi_r \int_0^{\bar{R}} \int_0^{1-R} f dL dR + \ \pi_l \int_0^{\bar{L}} \int_0^{1-L} f dL dR + \pi_H \left(\int_{\frac{\eta^* \bar{L}}{1-\eta^*}}^{\bar{R}} \int_0^{\frac{(1-\eta^*)R}{\eta^*}} f dL dR + \int_0^{\frac{\eta^* \bar{L}}{1-\eta^*}} \int_0^{\bar{L}} f dL dR \right)} \\ & + \frac{\pi_H \left(\int_{\frac{\eta^* \bar{L}}{1-\eta^*}}^{\bar{R}} \int_0^{\frac{(1-\eta^*)R}{\eta^*}} q_R f dL dR + \int_0^{\frac{\eta^* \bar{L}}{1-\eta^*}} \int_0^{\bar{L}} q_R f dL dR \right)}{\pi_r \int_0^{\bar{R}} \int_0^{1-R} f dL dR + \ \pi_l \int_0^{\bar{L}} \int_0^{1-L} f dL dR + \pi_H \left(\int_{\frac{\eta^* \bar{L}}{1-\eta^*}}^{\bar{R}} \int_0^{\frac{(1-\eta^*)R}{\eta^*}} f dL dR + \int_0^{\frac{\eta^* \bar{L}}{1-\eta^*}} \int_0^{\bar{L}} f dL dR \right)} \end{aligned} \tag{13}$$

The LHS is continuous, strictly increasing and $LHS \in [0, 1]$. The RHS is continuous, and as $\eta^* \to 0$, equations (11) and (12) imply that $\bar{R} \to 0$ and $\bar{L} \to 1$; therefore the $RHS \to \frac{\int_0^1 \int_0^{1-L} \frac{R}{R+L} f dL dR}{\int_0^1 \int_0^{1-L} f dL dR} > 0$. Similarly, as $\eta^* \to 1$, by equations (11) and (12) $\bar{R} \to 1$ and $\bar{L} \to 0$; therefore, $RHS \to \frac{\int_0^1 \int_0^{1-R} \frac{R}{R+L} f dL dR}{\int_0^1 \int_0^{1-R} f dL dR} < 1$. Therefore, there exists η^* that solves equation (13). To show the uniqueness we can take the derivative of the RHS of equation (13) with respect to η^* and evaluate it at the point at which $\eta^* = RHS(\eta^*)$. We have $\frac{dRHS}{d\eta^*} = \frac{\partial RHS}{\partial \eta^*} + \frac{\partial RHS}{\partial \bar{R}} \frac{d\bar{R}}{d\eta^*} + \frac{\partial RHS}{\partial \bar{L}} \frac{d\bar{L}}{d\eta^*}$, and using equation (11) and equation (12) we can show that for $\eta^* = RHS(\eta^*)$ we have $\frac{\partial RHS}{\partial \eta^*} = 0$, $\frac{\partial RHS}{\partial \bar{R}} = 0$, and $\frac{\partial RHS}{\partial \bar{L}} = 0$. Every time $\eta^* = RHS(\eta^*)$, the derivative $\frac{dRHS}{d\eta^*} = 0$; therefore, there is at most one solution. ■

Proof of Proposition 7

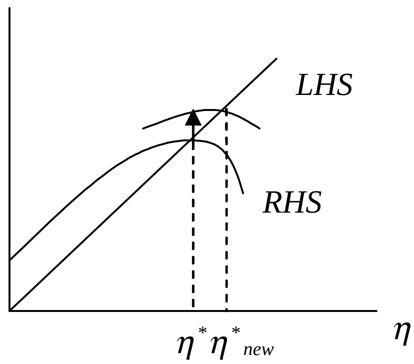
Equation (9) characterizes η_R^*. It does not depend on π_l; therefore, we can take the limit of equation (9) keeping π_r constant. We get $\lim_{\pi_H \to 1} \eta_R^* = E\left[\frac{R}{R+L}|R\right]$, and $\lim_{\pi_H \to 0} \eta_R^* \to 1$. The definition of λ_R, $\eta_R^* \equiv \frac{R}{R+\lambda_R}$, implies that $\lim_{\pi_H \to 0} \lambda_R = 0$; that is, the reports of the persuader become more extreme, and that $\lim_{\pi_H \to 1} \lambda_R = \bar{\lambda}_R = \frac{1-E[q_R|R]}{E[q_R|R]} R$. The analogous holds for P_l.

If we keep $\frac{\pi_H}{1-\pi_r}$ constant, the shape of the ambiguity area for P_l remains unchanged, which can be seen if we investigate the analog of equation (9) for P_l. Keeping $\frac{\pi_H}{1-\pi_r}$ constant implies that π_H increases as π_r decreases; therefore, equation (9) implies that the ambiguity area of P_r shrinks. That means that $\lim_{\pi_r \to 0} \lambda_R \to 0$, which means that the reports of P_r become more extreme. Now, I will show that η^* increases as π_r goes down.

η^* is determined by equation (13), where $\bar{R}$ and $\bar{L}$ are determined by equations (11) and (12). If we take the derivative of the RHS of (13) with respect to π_r and evaluate at η^*, we get

$$\begin{aligned} \frac{dRHS}{d\pi_r}|_{\eta^*} &=_{sign} \frac{1}{(1-\pi_r)} \int_0^{\bar{R}} \int_0^{\infty} \left(\frac{R}{R+L} - \eta^* \right) f dL dR + \\ &+ \left(\pi_r \int_0^{\infty} \left(\frac{\bar{R}}{\bar{R}+L} - \eta^* \right) f dL + \pi_H \int_0^{\frac{1-\eta^*}{\eta^*}\bar{R}} \left(\frac{\bar{R}}{\bar{R}+L} - \eta^* \right) f dL \right) \frac{d\bar{R}}{d\pi_r} \\ &= \frac{1}{(1-\pi_r)} \int_0^{\bar{R}} \int_0^{\infty} \left(\frac{R}{R+L} - \eta^* \right) f dL dR < 0, \end{aligned}$$

where the last equality comes from using equation (11). Recall that η^* is the point of intersection of the $LHS(\eta)$ and the $RHS(\eta)$ of equation (13) for the initial π_r, like in the picture below.



The fact that $\frac{dRHS}{d\pi_r} < 0$ implies that as π_r decreases the function $RHS\left(\eta\right)$ shifts up, which implies that the new $\eta^*_{new} > \eta^*$.

This implies that when π_r goes down, P_r is better-off, and P_l is worse-off. As π_r decreases, η^* and η^*_R for each R increase, which implies that for each R the persuader toward *Right* can induce a higher belief. Since the shape of the ambiguity area for P_l is not affected for big L, P_l can induce the same belief. However, the belief in the double ambiguity area is higher, and it is achieved for lower $\bar{L}_{new} < \bar{L}$ which means that for $L < \bar{L}$, P_l induces higher beliefs than before.

Showing that the utility of the decision maker increases requires some tedious algebra, which I omit here, but the result is intuitive, since it is more likely that the decision maker faces the honest expert. ■

Proof of Proposition 8

Step 1 For a given q_R, the expected utility of the decision maker with parameter θ is

$$\text{for } q_R < \theta : U\left(q_R\right) = \left(1-\pi\right)\int_0^{\infty}\left(\theta - q_R\right)g\left(N;z\right)dN + \pi\int_0^{\frac{R_\theta}{q_R}}\left(\theta - q_R\right)g\left(N;z\right)dN$$
$$+\pi\int_{\frac{R_\theta}{q_R}}^{\infty}\left(q_R - \theta\right)g\left(N;z\right)dN = \left(1-2\pi\right)\left(\theta - q_R\right) + 2\pi\left(\theta - q_R\right)G\left(\frac{R_\theta}{q_R};z\right),$$

$$\text{for } q_R > \theta : U(q_R) = \int_0^{\frac{R_\theta}{q_R}} (\theta - q_R)\, g(N;z)\, dN + \int_{\frac{R_\theta}{q_R}}^{\infty} (q_R - \theta)\, g(N;z)\, dN =$$
$$= 2(\theta - q_R)\, G\left(\frac{R_\theta}{q_R}; z\right) + (q_R - \theta).$$

Hence

$$E[U] = \int_0^{\theta} \left((1 - 2\pi)(\theta - q_R) + 2\pi (\theta - q_R)\, G\left(\frac{R_\theta}{q_R}; z\right)\right) dq_R + \int_\theta^1 \left(2(\theta - q_R)\, G\left(\frac{R_\theta}{q_R}; z\right) + (q_R - \theta)\right) dq_R.$$

Step 2 R_θ is such that the decision maker is indifferent between *Right* and *Left;* therefore, $\frac{dE[U]}{dz} = \frac{\partial E[U]}{\partial z} + \frac{\partial E[U]}{\partial R_\theta}\frac{dR_\theta}{dz} = \frac{\partial E[U]}{\partial z}$. Hence, we can look only at the direct effect of changing z.

We have

$$\frac{\partial E[U]}{\partial z} = \int_0^{\theta} 2\pi (\theta - q_R)\, G_z\left(\frac{R_\theta}{q_R}; z\right) dq_R + \int_\theta^1 2(\theta - q_R)\, G_z\left(\frac{R_\theta}{q_R}; z\right) dq_R.$$

I will prove in Step3 that $\bar{N} > \frac{R_\theta}{\theta}$. Using this, the second expression is positive since for $q_R \in (\theta, 1)$ we have $(\theta - q_R) < 0$ and $\frac{R_\theta}{q_R} \leq \frac{R_\theta}{\theta} < \bar{N}$. The first expression can be rewritten

$$\int_0^{\theta} 2\pi (\theta - q_R)\, G_z\left(\frac{R_\theta}{q_R}; z\right) dq_R =$$
$$= \int_0^{\frac{R_\theta}{\bar{N}}} 2\pi (\theta - q_R)\, G_z\left(\frac{R_\theta}{q_R}; z\right) dq_R + \int_{\frac{R_\theta}{\bar{N}}}^{\theta} 2\pi (\theta - q_R)\, G_z\left(\frac{R_\theta}{q_R}; z\right) dq_R,$$

where the fist expression is positive and the second is negative.

The fact that the mean of N is unaffected by changes in z implies that $\int_0^{\infty} G_z(N;z) = 0$, which in turn implies that there exists a set (a, b) with $a \geq \bar{N}$ such that

$$\int_{\frac{R_\theta}{\bar{N}}}^{\theta} \left(-G_z\left(\frac{R_\theta}{q_R}; z\right)\right) dq_R = \int_{\frac{R_\theta}{y}}^{\frac{R_\theta}{x}} G_z\left(\frac{R_\theta}{q_R}; z\right) dq_R.$$

Clearly $\frac{R_\theta}{x} < \frac{R_\theta}{\bar{N}}$, hence to complete the proof we need to show that

$$\int_{\frac{R_\theta}{y}}^{\frac{R_\theta}{x}} 2\pi\left(\theta - q_R\right) G_z\left(\frac{R_\theta}{q_R}; z\right) dq_R > \int_{\frac{R_\theta}{\bar{N}}}^{\theta} 2\pi\left(\theta - q_R\right)\left(-G_z\left(\frac{R_\theta}{q_R}; z\right)\right) dq_R.$$

But

$$\int_{\frac{R_\theta}{y}}^{\frac{R_\theta}{x}} 2\pi\left(\theta - q_R\right) G_z\left(\frac{R_\theta}{q_R}; z\right) dq_R > 2\pi\left(\theta - \frac{R_\theta}{x}\right)\int_{\frac{R_\theta}{y}}^{\frac{R_\theta}{x}} G_z\left(\frac{R_\theta}{q_R}; z\right) dq_R >$$

$$> 2\pi\left(\theta - \frac{R_\theta}{\bar{N}}\right)\int_{\frac{R_\theta}{y}}^{\frac{R_\theta}{x}} G_z\left(\frac{R_\theta}{q_R}; z\right) dq_R = 2\left(\theta - \frac{R_\theta}{\bar{N}}\right)\int_{\frac{R_\theta}{\bar{N}}}^{\theta}\left(-G_z\left(\frac{R_\theta}{q_R}; z\right)\right) dq_R >$$

$$> \int_{\frac{R_\theta}{\bar{N}}}^{\theta} 2\left(\theta - q_R\right)\left(-G_z\left(\frac{R_\theta}{q_R}; z\right)\right) dq_R.$$

Step 3 $\bar{N} > \frac{R_\theta}{\theta}$

Given the assumptions, the conditional p.d.f. of N given R is $g\left(N|R; z\right) = \frac{\frac{g(N;z)}{N}}{\int_R^1 \frac{1}{N} g(N;z) dN}$. The number of arguments in favor of *Right* for which the decision maker with parameter θ is indifferent between the alternatives, R_θ, is defined by $\theta = \eta^*_{R_\theta}$, and by the definition of $\eta^*_{R_\theta}$, one obtains

$$\begin{aligned}\theta &= \eta^*_{R_\theta} \geq E\left[\frac{R_\theta}{N}\right] = \frac{\int_{R_\theta}^1 \frac{R_\theta}{N}\frac{1}{N} g\left(N; z\right) dN}{\int_{R_\theta}^1 \frac{1}{N} g\left(N; z\right) dN} = \\ &= \frac{R_\theta E\left[\frac{1}{N^2}|N > R_\theta; z\right]}{E\left[\frac{1}{N}|N > R_\theta; z\right]} > R_\theta E\left[\frac{1}{N}|N > R_\theta; z\right] > \frac{R_\theta}{\bar{N}}\end{aligned}$$

Full revelation of information in the case of $z = \infty$ is a straightforward result. ■

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